3E1206

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B. Tech. III - Sem. (Main / Back) Exam., February - 2023

Automobile Engineering

3AE2 – 01 Advance Engineering Mathematics - I

AN, AG, AE, CE, CR, EC, EI, ME, MH, PT

Time: 3 Hours **Maximum Marks: 70**

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and three questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART – A

(Answer should be given up to 25 words only) **[10×2=20]**

All questions are compulsory

Q.1 Find the Laplace transform of -

$$f(t) = \begin{cases} \cos t & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases}$$

Q.2 What is unit step function?

Q.3 Find the Z-transform of sequences -

$$\{u_n\} = \{25, 10, 5, 3, 2, 1, 0, 5\} \quad -3 \le n \le 4$$

Q.4 Find the inverse Z-transform of $\log\left(\frac{z}{z+1}\right)$ by power series method.

Q.5 State Convolution Theorem for Fourier transform.

Q.6 Find the Fourier transform of $f(x) = \begin{Bmatrix} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{Bmatrix}$.

Q.7 Prove $E = 1 + \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$.

Q.8 By using Lagrange's formula, find x corresponding to y = 10 of following data -

x	10	15	17	20
y	3	7	11	14

Q.9 Find the first approximation value of x by Newton-Raphson method of $f(x) = xe^x - 2$ upto three decimal places.

Q.10 Write formula of Milne's Predictor Corrector Method.

PART – B

(Analytical/Problem solving questions) **[5×4=20]**

Attempt all five questions

Q.1 Find the inverse Laplace transform of $\frac{2s^2-1}{(s^2+1)(s^2+4)}$.

Q.2 Find Fourier sine and cosine transform of -

$$f(x) = \begin{Bmatrix} x & \text{for} & 0 < x \le 1 \\ 2 - x & \text{for} & 1 < x < 2 \\ 0 & \text{for} & x \ge 2 \end{Bmatrix}$$

Q.3 If $\bar{u}(z) = \frac{2z^2 + 3z + 4}{(z-3)^3}$, $|z| > 3$, then show that $u_1 = 2$, $u_2 = 21$ and $u_3 = 139$.

Q.4 Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using –

(i) Trapezoidal rule

(ii) Simpson 1/3 rule

Q.5 Given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$. Determine y (0.02) and y (0.04) by using modified Euler's method.

Q.6 By using Stirling formula, find u_{32} from the following data -

$u_{20} = 14.035$, $u_{25} = 13.674$, $u_{30} = 13.257$

$u_{35} = 12.734$ $u_{40} = 12.089$, $u_{45} = 11.309$

Q.7 Solve linear difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ given $u_0 = 1 = u_1$.

PART – C

(Descriptive/Analytical/Problem Solving/Design Questions) **[3×10=30]**

Attempt any three questions

Q.1 From the following table of values of x and y find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$ -

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.72	3.32	4.06	4.96	6.05	7.39	9.02

Q.2 If $\frac{dy}{dx} = x + y^2$ use Runge-Kutta method to find an approximate value of y for $x = 0.2$, given that $y = 1$ when $x = 0$. Use Laplace transform to solve.

Q.3 $(D^2 + 9)y = \cos 2t$, $y(o) = 1$, $y(\pi/2) = -1$.

Q.4 Obtain the Fourier transform of $f(x) = \begin{Bmatrix} x^2 & |x| \le a \\ 0 & |x| > a \end{Bmatrix}$. Hence evaluate

$$\int_0^{\infty} \cos\left(\frac{as}{2}\right) \frac{\left(a^2s^2 - 2\right)\sin as + 2as\cos as}{s^3} . ds$$

Q.5 Find $Z\{a^{|n|}\}$ and hence find $Z\left\{\left(\frac{1}{2}\right)^{|n|}\right\}$.
