

**4E1218**

Roll No. \_\_\_\_\_

Total No of Pages: **3****4E1218****B. Tech. IV-Sem. (Back) Exam., Oct.-Nov. - 2020****Electronic Inst. & Control Engg.****4EI2-01 Advanced Engineering Mathematics-II****EC, EI****Time: 2 Hours****Maximum Marks: 82****Min. Passing Marks: 29**[ersahilkagyan.com](http://ersahilkagyan.com)*Instructions to Candidates:*

*Attempt all ten questions from Part A, four questions out of seven questions from Part B and two questions out of five from Part C.*

*Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.*

*Use of following supporting material is permitted during examination. (Mentioned in form No. 205)*

1. NIL2. NIL**PART – A****(Answer should be given up to 25 words only)****[10×2=20]****All questions are compulsory**

Q.1 Write the necessary conditions for  $f(z)$  to be analytic.

Q.2 What is Isolated and Removable singularity?

Q.3 Find the points in the  $z$ -plane where the mapping  $w = f(z)$  fails to be conformal, when  $f(z) = z^4 - z^2$ .

Q.4 Evaluate  $\int_C \frac{dz}{z^2-1}$ , where  $C$  is the circle  $x^2+y^2=4$ .

Q.5 State Residue theorem.

Q.6 Compute the residue at each pole for the function –

$$F(z) = \frac{z^2}{z^2 - 2z + 2}$$

Q.7 Write the orthogonality of Legendre Polynomials.

Q.8 Find the value of  $J_{3/2}(x)$ .

Q.9 Define vector space with examples.

Q.10 What is Gram-Schmidt orthogonalization process.

## **PART – B**

**(Analytical/Problem solving questions)**

**[4×8=32]**

**Attempt any four questions**

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Q.1 Determine the analytic function  $w = u + iv$ , if  $u = e^{2x}(x \cos 2y - y \sin 2y)$ .

Q.2 Expand  $\frac{z-1}{z+1}$ , by Taylor's series about the point  $z = 0$ ;  $|z| < 1$ .

Q.3 Evaluate  $\int_0^{2\pi} \frac{d\theta}{5+4 \sin \theta}$  by contour integration.

Q.4 Express  $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$  in terms of Legendre's polynomial.

Q.5 Show that  $\int x J_0^2(x) dx = \frac{x^2}{2} [J_0^2(x) + J_1^2(x)] + c$ .

Q.6 Show that  $S_1$  is a vector subspace, where –

$$S_1 = \{(x_1, x_2, x_3) : x_1 + x_2 = x_3\}$$

Q.7 Use Gram Schmidt process to obtain an orthogonal basis from the basis set  $\{(1, 0, 1), (1, 1, 1), (1, 3, 4)\}$  of the Euclidean space  $\mathbb{R}^3$  with standard inner product.

## PART – C

(Descriptive/Analytical/Problem Solving/Design Questions) [2×15=30]

Attempt any two questions

- Q.1 (a) Show that under the transformation  $w = \frac{z-1}{z+1}$ , real axis in the  $z$ -plane is mapped into the circle  $|w| = 1$ . What portion of the  $z$ -plane corresponds to the interior of the circle?
- (b) Find the bilinear transformation which maps the points  $z = 0, -i, -1$  into  $w = i, 1, 0$
- Q.2 (a) Expand the function  $\frac{1}{(z-1)(z-2)}$  for –
- (i)  $|z| < 1$  (ii)  $1 < |z| < 2$  (iii)  $|z| > 2$  (iv)  $0 < |z - 1| < 1$
- (b) Evaluate the integral  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ ;  $|z|=3$
- Q.3 Construct an orthonormal basis from  $E^4$  with  $\frac{1}{2}(1,1,1)$  as the starting basis.
- Q.4 Show that  $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)^2} = \frac{\pi(a+2b)}{2ab^3(a+b)^2}$  where  $b > 0$  and  $c > 0$
- Q.5 (a) Prove that  $\int_{-1}^{+1} (P_n')^2 dx = n(n+1)$
- (b) Prove that  $\frac{d}{dx} [J_n^2(x) + J_{n+1}^2(x)] = 2 \left[ \frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x) \right]$