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B. Tech. I - Sem. UD (Main / Back) Exam., Jan. - 2020

IFY2 – 01 Engineering Mathematics - I

Admitted Batch: 2018 – 19 & 2019 - 20

Time: 3 Hours **Maximum Marks: 100**

Min. Passing Marks: 33

Instructions to Candidates:

PART - A *: Short answer questions (up to 25 words) 10 × 2 marks =* ***20 marks.***

All ten questions are compulsory.

PART - B *: Analytical/Problem Solving questions (up to 100 words) 6×5 marks=****30 marks.***

Candidates have to answer six questions out of ***eight.***

PART - C *: Descriptive/Analytical/Problem Solving questions 5×10 marks =****50 marks.***

Candidates have to answer five questions out of ***seven.***

1. NIL 2. NIL

PART – A

Q.1 What are the value of integral $\int_0^\infty e^{-x^2}\, dx$?

Q.2 Write the formula for surface area of solid of revolution when the revolution is about y – axis.

Q.3 Show the sequence $\{x_n\}$, where $x_n = \frac{2n-72}{3n+23}$ converges to $\frac{2}{3}$.

Q.4 Find the value of a_0 for the function $f(x) = |x|$ in the interval $(-\pi, \pi)$.

Q.5 Show that $\lim\limits_{(x,y)\to(0,0)} \frac{2x-y}{x^2+y^2}$ does not exist.

Q.6 State the necessary and sufficient condition for the minimum of a functions $f(x, y)$.

Q.7 If $u = e^{xyz}$, find $\frac{\partial^2 u}{\partial y\, \partial z}$.

Q.8 Evaluate $\int_0^b \int_0^x xy\, dxdy$.

Q.9 State the Gauss divergence theorem.

Q.10 If $\vec{F} = xy^2\hat{i} + 2x^2yz\hat{j} - 3yz^2\hat{k}$, find div $\vec{F}$ at the point $(1, -1, 1)$.

PART – B

Q.1 Examine the convergence of the series $u_n = \frac{\sqrt{n}}{3n-1}$.

Q.2 Let $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 - y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$,

then show that the function f is not differentiable at the origin.

Q.3 Find the Fourier series for the function $f(x) = x, \quad -\pi < x < \pi$.

Q.4 State Euler's theorem and if $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then prove that -

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

Q.5 Find the points where the function $x^3 + y^3 - 3axy$ has maximum or minimum value.

Q.6 Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dxdy$ by changing the order of integration.

Q.7 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then prove that -

$$\operatorname{div} r^n\vec{r} = (n + 3)\, r^n.$$

Hence, show that $r^n\vec{r}$ will be solenoidal if n = -3.

Q.8 Prove that equation:

$$\int_0^{\pi/2} \frac{d\theta}{\sqrt{(a\cos^4\theta + b\sin^4\theta)}} = \frac{\left\{\Gamma\frac{1}{4}\right\}^2}{4(ab)^{\frac{1}{4}}\sqrt{x}}$$

PART – C

Q.1 Use beta and gamma functions, to evaluate -

(a) $\int_0^\infty \frac{x}{1 + x^6}\, dx$

(b) $\int_0^1 \sqrt{\left(\frac{1-x}{x}\right)}\, dx$

Q.2 Test the convergence of the series:

$$1 + \frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \ldots\ldots$$

Q.3 Find the Fourier series for the function $f(x) = x\sin x$, $-\pi < x < \pi$ and deduce that :

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \ldots\ldots.$$

Q.4 If $u = f(r)$, where $r^2 = x^2 + y^2$, then prove that -

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$

Q.5 Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2 + y^2}\, dxdy$ by changing into polar coordinates.

Q.6 Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dxdydz$

Q.7 Verify Stoke's theorem for the vector function $\vec{F} = x^2\hat{i} + xy\hat{j}$, where C is the perimeter of the square in xy – plane whose sides are along the lines x = 0, y = 0, x = a and y = a.
