B.Tech. II Semester (Main) Exam 2022

2FY2-01 Engineering Mathematics-II

2E3201

Time: 3 Hours

Maximum Marks: 70

Part-A (All Ten Questions)

Q.1 Let v_1, v_2 and v_3 be the first, second and third column vectors, respectively, of the matrix

$$A = \begin{pmatrix} 2 & 1 & 7 \\ 1 & 0 & 2 \\ -1 & 5 & 13 \end{pmatrix}.$$

What can we conclude about rank(A) from the observation $2v_1 + 3v_2 - v_3 = 0$?.

Q.2 Suppose the system $AX = B$ is consistent and A is a 5×8 matrix and rank(A) = 3. How many parameters does the solution of the system have ?

Q.3 State Cayley-Hamilton Theorem.

Q.4 Write the non-linear first order Bernoulli equation.

Q.5 Define Exact first order differential equation.

Q.6 Write the Euler-Cauchy differential equation.

Q.7 Write Clairaut's type differential equation.

Q.8 Write Bessel's differential eqiation.

Q.9 Write the Charpit's equations for the first order partial differential equation $f(x, y, z, p, q) = 0$.

Q.10 Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$.

$10 \times 2 = 20$

Part-B (All Five Questions)

Q.1 Find the values of λ for which the equations

$$\begin{aligned} (\lambda - 1)x + (3\lambda + 1)y + 2\lambda z &= 0 \\ (\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z &= 0 \\ 2x + (3\lambda + 1)y + 3(\lambda - 1)z &= 0 \end{aligned}$$

Q.2 Solve the differential equation

$$(2y^3xe^y + y^2 + y)dx + (y^3x^2e^y - xy - 2x)dy = 0.$$

Q.3 Solve: $y = 2px + yp^2$; where $p = \frac{dy}{dx}$.

Q.4 Solve: $(D^2 - 4D + 13)y = 18e^{2x}\sin 3x$; where $D \equiv \frac{d}{dx}$.

Q.5 Find the general solution of the partial differential equation

$$(3 - 2yz)p + x(2z - 1)q = 2x(y - 3), \text{ where } p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

Part-C (Any Three Questions) **5 × 4 = 20**

Q.1 Examine whether the matrix

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

is diagonalizable. If so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix.

Q.2 Find the general solution of the differential equation

$$(D^2 + 4D + 4)y = e^{-2x}\sin x, \ D \equiv \frac{d}{dx}$$

using the method of variation of parameters.

Q.3 Find the power series solution of $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ about $x = 0$.

Q.4 Find the complete integral of the partial differential equation

$$p^2q^2 = 9p^2y^2(x^2 + y^2) - 9x^2y^2.$$

Q.5 Solve the following equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.