1E3101

Total No. of Questions : 22

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Roll No. : ....

1E3101

B.Tech. I sem(Main/Back) Exam 2024

1FY2-01 / Engineering Mathematics-I

Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates :

Attempt all ten questions from Part-A, five questions out of seven questions from Part-B and three questions out of five questions from Part-C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used / calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in Form No. 205)

1. ....   2. ....

PART - A

(Answer should be given up to 25 words only)

All questions are compulsory

Q.1. What is the value of integral $\int_0^\infty e^{-x^2} dx$?

Q.2. Write the formula of surface area of solid of revolution when the revolution is about x-axis.

Q.3. What do you mean by convergence of a sequence?

Q.4. Find whether the following series is convergent or not?

$$\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\dots$$

Q.5. State Parseval's theorem.

Q.6. Find the value of a_0 for the function $f(x)=|x|$ in the interval $(-\pi,\pi)$.

Q.7. State the necessary and sufficient conditions for the minimum of a function $f(x,y)$.

Q.8. Find the gradient of $f(x,y,z)=x^2y^2+xy^2-z^2$ at (3, 1, 1).

Q.9. Evaluate $\int_0^b\int_0^x xy\,dx\,dy$.

Q.10. State the Gauss Divergence theorem.

PART - B

(Analytical/Problem solving questions)

Attempt any five questions

Q.1. Use beta and gamma functions, to evaluate :

$$\int_0^\infty \frac{x^2(1+x^4)}{(x+x)^{10}}dx.$$

Q.2. Expand sin x in the powers of $(x-\pi/2)$ using Taylor's series.

Q.3. Find Fourier series of x^2 in $(-\pi,\pi)$, and use Parseval's identity to prove :

$$\frac{\pi^4}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+...$$

Q.4. If $u=e^{xyz}$, then show that :

$$\frac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$$

Q.5. Whether the fluid motion given by $V=(y+z)i+(z+x)j+(x+y)k$ is incompressible or not?

Q.6. Change the order of integration and hence evaluate :

$$\int_0^1\int_{e^x}^{e} \frac{1}{\log y}dxdy.$$

Q.7. Evaluate $\int_1^2\int_1^z\int_0^{yz} (xyz)dxdydz.$

PART - C

(Descriptive/Analytical/Problem Solving/Design question)

Attempt any three questions

Q.1. Use beta and gamma functions, to evaluate :

(a) $\int_0^\infty \frac{x}{1+x^6}dx.$

(b) $\int_0^1 \sqrt{\left(\frac{1-x}{x}\right)}\,dx.$

Q.2. Find the Fourier series expansion of the following periodic function with period 2π.

$$f(x)=\begin{cases} -1, & -\pi<x<0 \\ 0, & x=0 \\ 1, & 0<x<\pi \end{cases}$$

Hence, show that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-....=\frac{\pi}{4}$.

Q.3. Use Lagrange's method to find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2+y^2+z^2=1$.

Q.4. If $u=f(r)$, where $r^2=x^2+y^2$, then prove that :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r).$$

Q.5. Verify Green's theorem for $\int_C\left[(xy+y^2)dx+x^2dy\right]$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.