

1E3101**1E3101**

B.Tech. I Sem. (Main) Examination, April/May - 2022
1FY2-01 Engineering Mathematics-I

Maximum Marks : 70

Time : 3 Hours

Instructions to Candidates:

Attempt all ten questions From Part A, five Questions out of seven questions from Part B and three questions out of five questions from Part C.
Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination (As mentioned in form No. 205).

Part - A

(Answers should be given up to 25 words only)

(10×2=20)

All questions are compulsory.

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1. Define Beta function.
2. Write Euler's formula for a Fourier series.
3. Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x=2, y=1$?
4. Consider a spatial curve in three-dimensional space given in parametric form by $x(t) = \cos t, y(t) = \sin t, z(t) = \frac{2}{\pi}t, 0 \leq t \leq \frac{\pi}{2}$. The length of the curve is.....
5. In the Taylor series expansion of e^x about $x=2$, the coefficient of $(x-2)^4$ is
6. Define the convergence of a power series.
7. The Directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point $P=(1,1,2)$ in the direction of the vector $\vec{a} = 3\hat{i} - 4\hat{j}$ is

8. Curl of vector $\vec{V}(x, y, z) = 2x^2\hat{i} + 3z^2\hat{j} + y^3\hat{k}$ at $x = y = z = 1$ is
9. Velocity vector of a flow field is given as $\vec{V}(x, y, z) = 2xy\hat{i} - 3x^2z\hat{j}$. The vorticity vector at $(1, 1, 1)$ is
10. The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

Part - B

(Analytical/Problem solving questions)

Attempt any five questions:

(5×4=20)

1. Evaluate the following integrals:

i) $\int_0^{\pi} x^4 e^{-x^4} dx$

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ii) $\int_0^{\pi/2} \sin^6 \theta \cos^7 \theta d\theta$.

2. The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.

3. Test the convergence/divergence of the series.

$$\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \dots$$

4. Find the Fourier series expansion of the following periodic function with period 2π :

$$f(x) = \begin{cases} \pi + x, & \text{if } -\pi < x < 0 \\ 0, & \text{if } 0 \leq x < \pi \end{cases}$$

5. Consider the function:

$$f(x, y) = \sqrt{\frac{e^{\sin(x)}}{x^{2014} + \sqrt{x^{2012} + 1}}} + \cos(xy). \text{ Find the second partial derivative } \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right).$$

6. A scalar potential ϕ has the gradient $\nabla \phi = yz\hat{i} + xz\hat{j} + zy\hat{k}$. Evaluate the integral $\int_C \nabla \phi \cdot d\vec{r}$ on the Curve $C: \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, if the curve C is parameterised as follows :
 $x = t, y = t^2, z = 3t^2, 1 \leq t \leq 3$.

7. Find the area of the region R in the xy-plane enclosed by the circle $x^2 + y^2 = 4$, above the line $y=1$, and below the line $y = \sqrt{3}x$.

Part - C

(Descriptive/Analytical/Problem solving/Design Questions)

(3×10=30)

Attempt any three questions.

1. If $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2-x), & 1 < x < 2 \end{cases}$ using half range cosine series, show that

$$\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

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2. Show that $\text{div}(\text{grad } r^n) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$. Hence, show that

$$\nabla^2\left(\frac{1}{r}\right) = 0.$$

3. The pressure P at any point (x, y, z) in space is $P = 400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.

4. Find the work done by a force $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + z)\hat{k}$ in moving a particle from $P(0, 1, -1)$ to $Q\left(\frac{\pi}{2}, -1, 2\right)$.

5. Apply stoke's theorem to find the value of $\int_C (y dx + z dy + x dz)$. Where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.