

SYLLABUS

UNIT 1

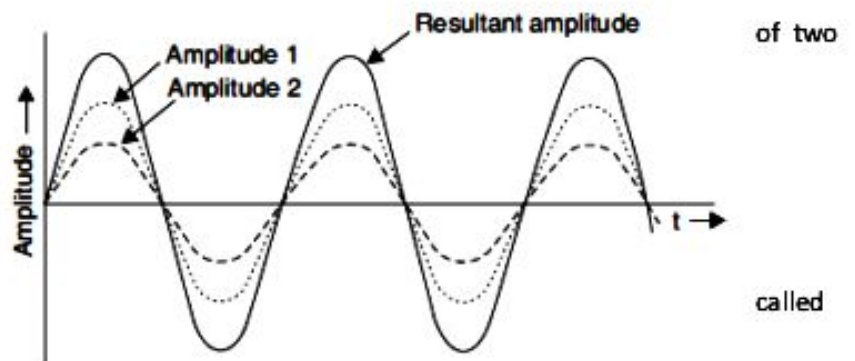
WAVE OPTICS

- ✓ **NEWTON'S RINGS**
- ✓ **MICHELSON'S INTERFEROMETER**
- ✓ **FRAUNHOFER DIFFRACTION FROM A SINGLE SLIT**
- ✓ **DIFFRACTION GRATING:**
 - **CONSTRUCTION,**
 - **THEORY AND SPECTRUM**
- ✓ **RESOLVING POWER AND RAYLEIGH CRITERION FOR LIMIT OF RESOLUTION**
- ✓ **RESOLVING POWER OF DIFFRACTION GRATING**
- ✓ **X-RAY DIFFRACTION AND BRAGG'S LAW**

INTERFERENCE OF LIGHT

When two or more light wave superimpose to each other and due to this resultant amplitude and intensity gets modified, this modification of intensity obtained by superposition light wave is called INTERFERENCE.

At some points, where intensity is maximum, interference is called CONSTRUCTIVE (we obtain BRIGHT FRINGES) and at some points, where intensity is minimum, interference is called DESTRUCTIVE (OBTAINED DARK FRINGES).

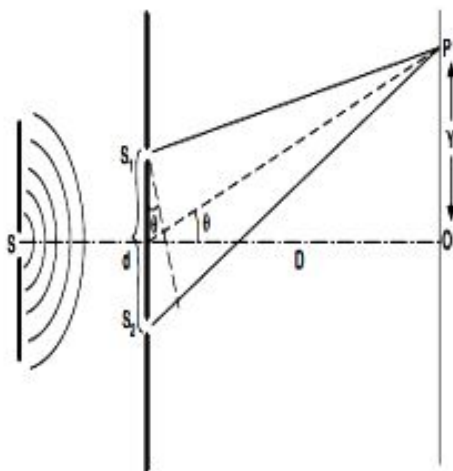


Since we know:

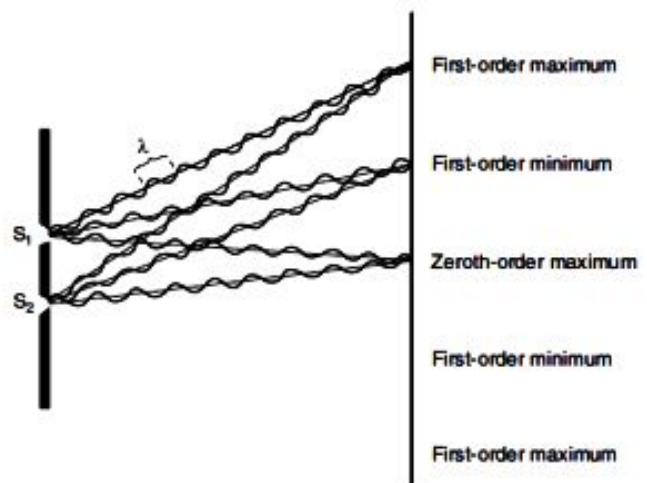
$$A = A_1 + A_2 + A_3 + \dots + A_n$$

$$I = (A_1 + A_2 + A_3 + \dots + A_n)^2$$

RESULTS OF YOUNG DOUBLE SLITS EXPERIMENT :



(FIGURE – A)



(FIGURE – B)

1. INTENSITY $I = R^2 = (A_1 + A_2 + 2 A_1 A_2 \cos \delta)$

Here A_1 = Amplitude of wave 1

A_2 = Amplitude of wave 2

δ = Phase difference of wave 1 and 2.

2. FRINGE WIDTH :

Since PATH DIFFERENCE

$$\Delta = \frac{x d}{D}$$

Here

x = Distance of fringe from central fringe

d = Distance between two slits

D = Distance between slits and screen.

1. FOR BRIGHT FRINGE :

$$\Delta = n\lambda$$

$$\frac{x_n d}{D} = n\lambda$$

$$x_n = \frac{n\lambda D}{d}$$

$$x_{n+1} - x_n = \frac{\lambda D}{d} \quad \text{----- (FRINGE WIDTH)}$$

2. FOR DARK FRINGE :

$$\Delta = (2n + 1) \frac{\lambda}{2}$$

$$\frac{x_n d}{D} = (2n + 1) \frac{\lambda}{2}$$

$$x_n = \frac{(2n+1)\lambda D}{2d}$$

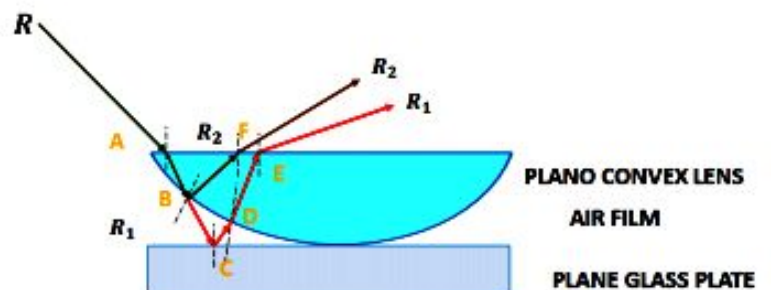
$$x_{n+1} - x_n = \frac{\lambda D}{d} \quad \text{----- (FRINGE WIDTH)}$$

CONDITION FOR SUSTAINED INTERFERENCE:

1. Light wave should be coherent
2. Should have same frequency
3. Should have same wavelength
4. Should have approximately same amplitude
5. Separation between two slits should be very small
6. Separation between screen and slits should be large
7. Plane of polarization should be same

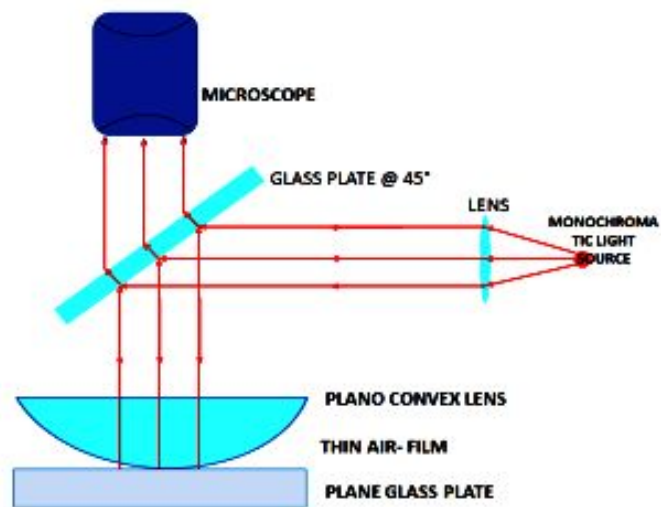
NEWTON'S RING:

Newton's ring are a series of concentric circular rings (Bright & Dark) which appears about the point of contact between a glass plate and a plano convex lens when illuminated with monochromatic light.



EXPERIMENTAL SET UP: When a Plano convex lens of large focal length is placed on a plane glass plate with its convex surface in contact with glass plate. An air film of varying thickness being prepared between the lower surface of lens and upper surface of glass plate. At the point of contact air film thickness will be zero.

When a beam of light is made to fall normally on it, concentric rings are observed with the help of microscope. Each ring is of circular shape with concentric nature are called as **NEWTON'S RINGS**.



DETERMINATION OF THICKNESS OF AIR FILM BY USING N.R. EXPERIMENT:

The path difference produced between the waves reflected by the wedge-shaped air film is

$$\Delta = 2\mu t \cos(r + \theta) + \frac{\lambda}{2} \quad \text{-----(1)}$$

As shown in figure from the Pythagoras Theorem on triangle CPN

$$R^2 = (CN)^2 + r_n^2 \quad \text{----- (2)}$$

$$\text{Since } CN = R - NO$$

$$CN = R - t_n$$

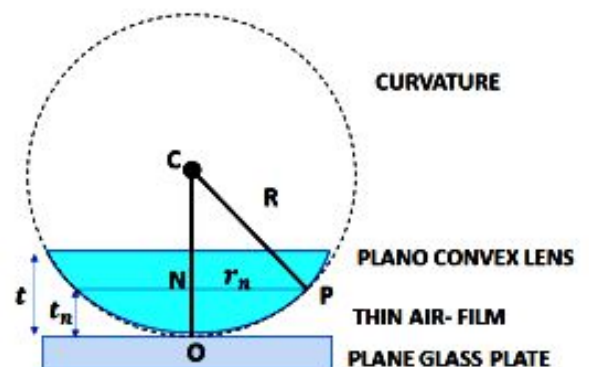
$$R^2 = (R - t_n)^2 + r_n^2$$

$$r_n^2 = -t_n^2 + 2Rt_n$$

$$\text{Since } t_n \ll R \text{ so we can say } t_n^2 \approx 0$$

$$\text{So } r_n^2 = 2Rt_n$$

$$t_n = \frac{r_n^2}{2R} \quad \text{----- (3)}$$



Which gives the thickness of air film formed (between plano convex lens and plane glass plate)to the corresponding Newton's Ring.

DIAMETER OF BRIGHT RINGS:

The path difference in reflected rays R_1 and R_2

$$\Delta = 2\mu t \cos(\theta) + \frac{\lambda}{2}$$

so for n^{th} order of bright fringe

$$2\mu t_n \cos \theta + \frac{\lambda}{2} = n\lambda \quad \text{----- (4)}$$

$$2\mu t_n \cos \theta = (2n - 1) \frac{\lambda}{2}$$

If $\theta = 0$ so $\cos \theta = 1$

$$\therefore 2\mu t_n = (2n - 1) \frac{\lambda}{2} \quad \text{----- (5)}$$

We know that $t_n = \frac{r_n^2}{2R}$ therefore putting the value of t_n in equation 5.

$$2\mu \frac{r_n^2}{2R} = (2n - 1) \frac{\lambda}{2} \quad \text{----- (6)}$$

$$r_n = [(2n - 1) \frac{\lambda R}{2\mu}]^{1/2} \quad \text{----- (7)}$$

for air $\mu=1$

$$\therefore r_n = [(2n - 1) \frac{\lambda R}{2}]^{1/2} \quad \text{----- (8)}$$

Or $D_n = 2r_n = 2 [(2n - 1) \frac{\lambda R}{2\mu}]^{1/2} \quad \text{----- (9)}$

DIAMETER OF DARK RINGS :

The path difference in reflected rays R_1 and R_2

$$\Delta = 2\mu t \cos(\theta) + \frac{\lambda}{2}$$

For dark rings $\Delta = (2n + 1) \frac{\lambda}{2}$

$$2\mu t_n \cos(\theta) + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2} \quad \text{----- (10)}$$

$$2\mu t_n \cos \theta = n\lambda$$

If $\theta = 0$ then $\cos\theta = 1$ and if $r = 0$ then $\cos r = 1$

$$2\mu t_n = n\lambda \quad \text{-----(13)}$$

But we know $t_n = \frac{r_n^2}{2R}$

$$\therefore 2\mu \frac{r_n^2}{2R} = n\lambda \quad \text{-----(14)}$$

$$\text{or } r_n^2 = \frac{n\lambda R}{\mu}$$

$$\text{or } r_n = \left[\frac{n\lambda R}{\mu} \right]^{1/2} \quad \text{-----(15)}$$

$$\text{or } D_n = \left[\frac{4n\lambda R}{\mu} \right]^{1/2} \quad \text{-----(17)}$$

$$\text{or for air } \mu = 1 \quad \therefore D_n = [4n\lambda R]^{1/2} \quad \text{-----(18)}$$

DETERMINATION OF WAVELENGTH OF MONOCHROMATIC LIGHT USING NEWTON'S RING EXPERIMENT:

Since we know: Diameter of n^{th} dark ring is

$$D_n = [4n\lambda R]^{1/2}$$

$$\text{Or } D_n^2 = [4n\lambda R] \quad \text{-----(19)}$$

and for $(n + p)^{th}$ dark ring

$$D_{n+p}^2 = [4(n + p)\lambda R]$$

$$\text{Now } D_{n+p}^2 - D_n^2 = [4(n + p)\lambda R] - [4n\lambda R]$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

MICHELSON INTERFEROMETER:

An interferometer is a device that can be used to measure length or changes in length with great accuracy by means of interference fringes. The basic principle of this was given by A.A. MICHELSON.

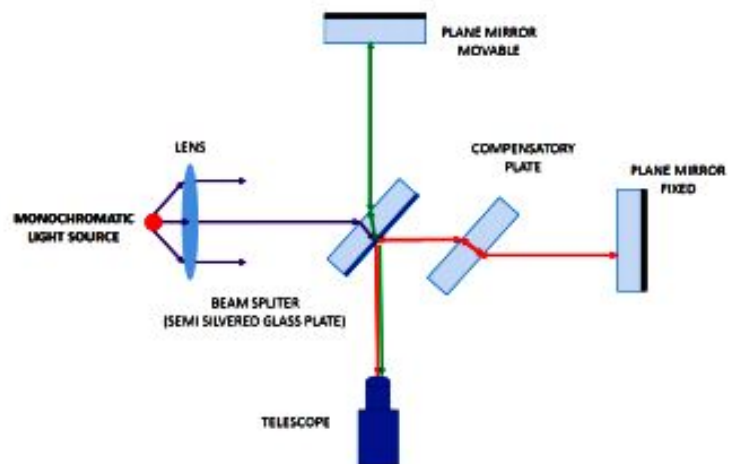
EXPERIMENTAL SETUP:

The essential parts of Michelson Interferometer are set of two identical same mirror M_1 and M_2 (highly polished), a plane glass plate that is semi silvered (as beam splitter) one another glass plate (as compensatory plate), a monochromatic light source, and a telescope.

The experimental setup is shown in figure in which both the mirrors M_1 and M_2 are kept at 90° angle (perpendicular to each other) and a beam splitter kept between them as shown.

WORKING:

When a light wave from a monochromatic light source, incident at 45° on to beam splitter, light wave splits in two part. One wave reflects to the mirror M_1 and other transmitted towards the mirror M_2 . Wave 2 passes through compensatory plate and incident on mirror M_2 then it reflects to the same path.

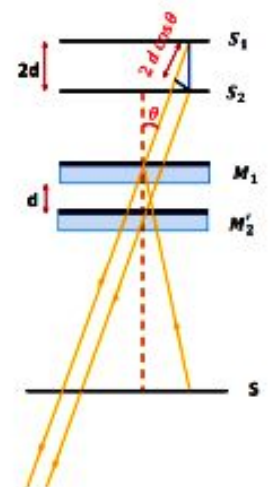


Wave 1 directly incident to the mirror M_1 and reflect to the same path. These two waves $1'$ and $2'$ now reaches to the point on beam splitter at same time. Due to these waves generated from a single wave and zero path difference, will show the phenomena of INTERFERENCE and will produce interference fringes.

ROLE (WORKING) OF COMPENSATORY PLATE:

The wave reflected at M_1 crosses the glass plate (beam splitter) twice while the other wave does not pass through it even once. So an extra optical path $2(\mu-1)t$ is introduced in wave 1 where ' t ' is the thickness of the plate and μ is refractive index.

It is essential to compensate this difference we have to introduce a glass plate with same thickness ' t ' and refractive index ' μ ' to the path of wave 2. So, the extra path difference can nullify.



FORMATION OF FRINGES AND SHAPE OF FRINGES

A virtual image of mirror M_2 is formed between the M_1 and beam splitter near by the mirror M_1 due to the reflection at beam splitter. In this way path difference of wave 1 and wave 2 becomes zero. A thin film of air is formed between M_1 and M_2' . So the fringes produced due to this thin air film. These fringes can be straight, circular or parabolic in shape. These shapes depend on the path difference and the angle between the mirrors M_1 and M_2 .

CIRCULAR FRINGES:

When mirror M_2 and M_1 are exactly perpendicular to each other, then circular fringes are formed. Let S_1 and S_2 be virtual images of the source due to M_1 and M_2' . Let d be the distance between M_1 and M_2' or the air film so the distance between S_1 and S_2 will be $2d$.

The path difference between light waves coming from S_1 and S_2 will be $2d \cos \theta$. When one light wave is reflected from denser medium to rarer medium, an additional path difference of $\lambda/2$ will be produced so the path difference

$$\Delta = 2d \cos \theta + \frac{\lambda}{2}$$

FOR MAXIMA:

$$2d \cos \theta + \frac{\lambda}{2} = n\lambda$$

$$\text{OR} \quad 2d \cos \theta = (2n - 1) \frac{\lambda}{2}$$

This equation when plotted will give a curve with same locus and this will be in circular shape.

DETERMINATION OF WAVELENGTH OF MONOCHROMATIC LIGHT USING MICHELSON INTERFEROMETER:

To determine the wavelength of light, adjust Michelson Interferometer in such a way that circular fringes are observed. Now move the mirror M_2 with the help of micrometer screw. The fringes cross the field of view. When M_2 is moved $\lambda/2$ then an additional path difference of λ will be introduced, for one fringe shift, the mirror moves through a distance equal to half wave length. The number of fringes that cross the field of view is counted.

Since we know the condition of dark fringe at the center is

$$2d \cos \theta = n\lambda$$

For central fringe $\theta = 0$ so $\cos 0 = 1$

$$\therefore 2d = n\lambda$$

Adding λ both sides

$$2d + \lambda = n\lambda + \lambda$$

$$2 \left[d + \frac{\lambda}{2} \right] = (n + 1)\lambda$$

It is clear that when $d \rightarrow d + \lambda/2$ then $n \rightarrow (n + 1)$

In other words, on moving mirror M1 by a distance

Let n be the number of fringes that crosses the center of the view

Then

$$x = \frac{n\lambda}{2}$$

Or $\lambda = \frac{2x}{n} \dots\dots\dots(1)$

DETERMINATION OF DIFFERENCE IN CLOSE WAVELENGTH RESOLUTION OF TWO SPECTRAL LINES:

A source of two closely spaced spectral lines λ_1 and λ_2 is used to determine the difference in these wavelengths.

Firstly, we have to adjust Michelson Interferometer to produce circular fringes. Pattern is now adjusted to obtain maximum distinctness. This is possible when the n th order fringe of longer wavelength coincides with $(n+1)^{\text{th}}$ order of shorter wavelength

$$2d \cos \theta = n \lambda_1$$

For λ_2 $2d \cos \theta = (n + 1)\lambda_2$

Now mirror M_1 is moved through distance x so that pattern of maximum distinctness is again obtained i.e. m fringes of λ_1 crosses the $(n+1)$ fringes of λ_2 wavelength.

$$\therefore x = \frac{n\lambda_1}{2} \quad \therefore n = \frac{2x}{\lambda_1} \dots\dots\dots(1)$$

and $x = \frac{(n+1)\lambda_2}{2}$

$$\text{or} \quad n + 1 = \frac{2x}{\lambda_2} \quad \text{-----}(2)$$

Subtracting equation 2 and 1

$$n + 1 - n = \left(\frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} \right) \quad \text{-----}(3)$$

$$1 = 2x \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right)$$

$$1 = 2x \left(\frac{\lambda_1 - \lambda_2}{\lambda_2 \lambda_1} \right)$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_2 \lambda_1}{2x}$$

$$\text{And} \quad \lambda_1 \lambda_2 = \lambda_{av}^2 = \left(\frac{\lambda_1 + \lambda_2}{2} \right)^2$$

$$\text{Therefore} \quad \lambda_1 - \lambda_2 = \frac{\lambda_{av}^2}{2x} \quad \text{-----}(4)$$

DIFFRACTION OF LIGHT

The bending of light around an obstacle or slit and spreading out in the geometrical shadow is called diffraction.

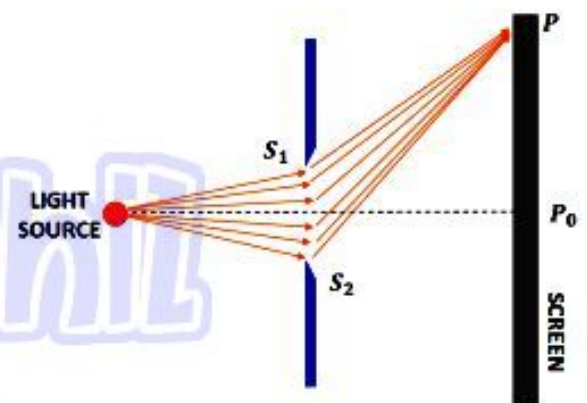
Diffraction of light depends on

- Size of obstacle (a)
- Wavelength of light (λ)

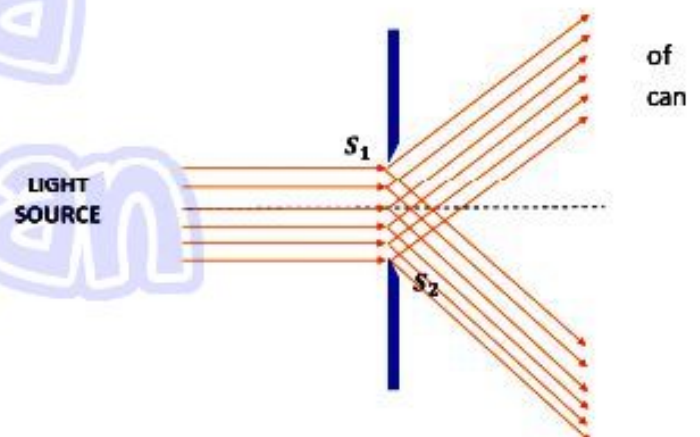
Diffraction is of two types:

1. Fresnel's Diffraction
2. Fraunhofer's Diffraction

1. FRESNEL'S DIFFRACTION :- In this type of diffraction source and screen are at a finite distance from diffracting element. The incident wave front may be circular or cylindrical but should not be plane. In this theoretical analysis is much complex.



2. FRAUNHOFER'S DIFFRACTION :- In this kind of diffraction source and screen are kept at infinite distance and observed by using a lens. The theoretical analysis is simple, the pattern is the image of the source.

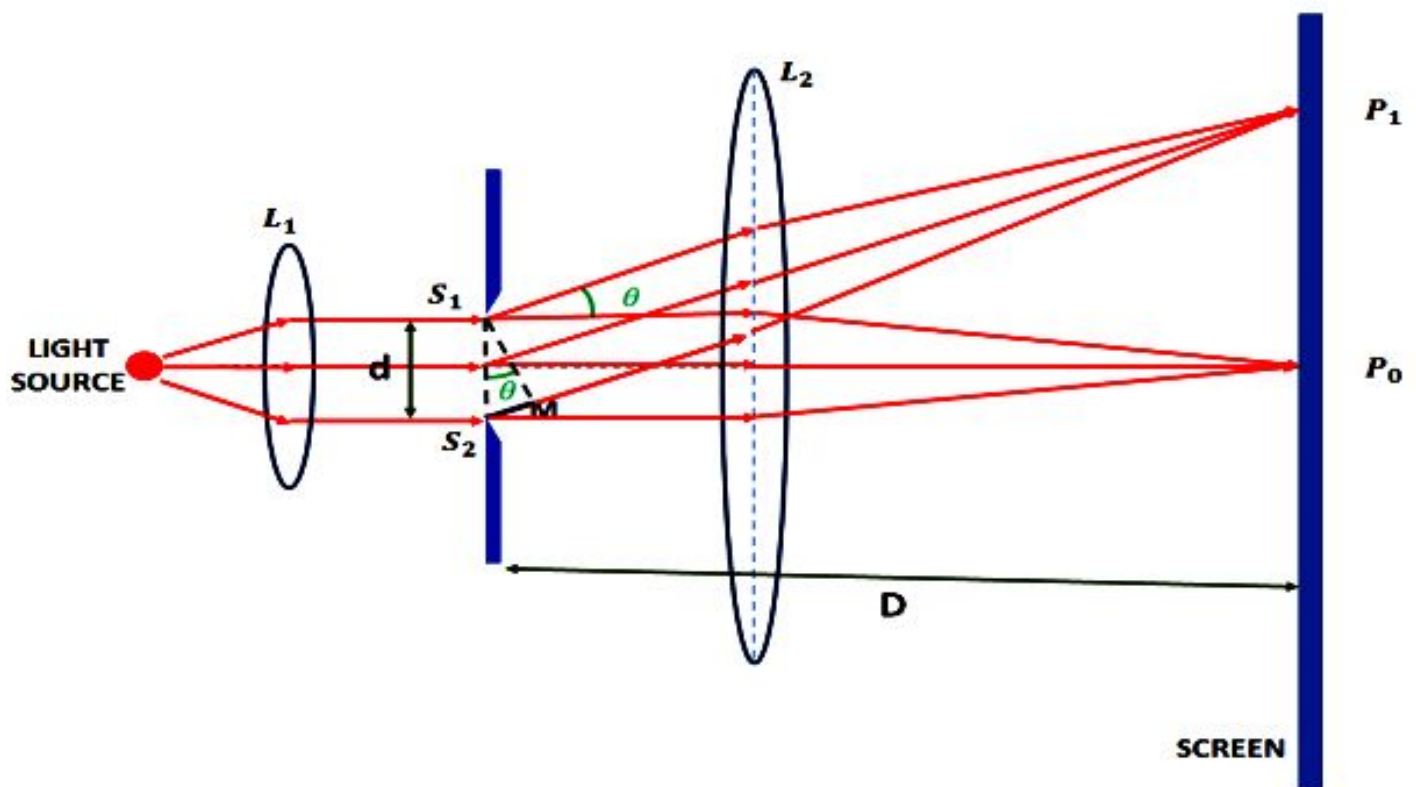


FRAUNHOFER'S DIFFRACTION AT SINGLE

SLIT:-

QUALITATIVE DESCRIPTION:

As we know a slit is a rectangular aperture whose length is large compared to its width. As shown in the figure, a slit S_1S_2 with width a . A wave front originates from the source S which, after refraction through the lens, becomes plane. Lens L_2 produces an image P_0 on the screen according to geometrical optics. Light is obtained in different directions on the screen. One of these images P is kept in consideration, which is basically a diffracted light wave, and images are the diffraction pattern. The path difference between the secondary wavelets focusing at point P and diffracted at an angle θ is S_2K .



Therefore $\Delta S_1 S_2 K$

$$\sin \theta = \frac{s_2 k}{s_1 s_2}$$

$$s_2 k = s_1 s_2 \sin \theta$$

$$s_2 k = a \sin \theta$$

QUANTITATIVE DESCRIPTION: -

$$s_2 k = a \sin \theta$$

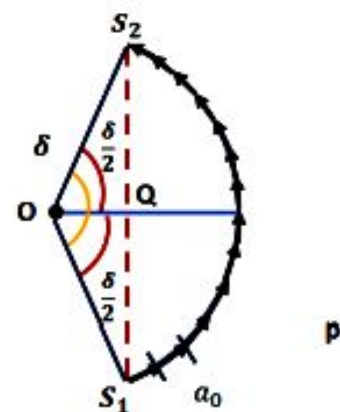
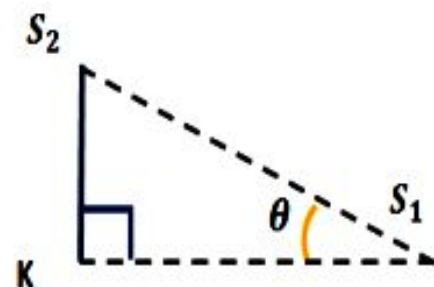
$$\Delta = a \sin \theta \quad \text{and} \quad \delta = \frac{2\pi}{\lambda} \times \Delta$$

$$\delta = \frac{2\pi}{\lambda} \times a \sin \theta = 2\alpha \quad (\text{LET})$$

Suppose the wave front observed from slit $S_1 S_2$ is an area as shown in figure and is divided into p smaller part with same width a_0 .

As we know:

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$



$$\delta = \frac{S_1 S_2}{OS_2}$$

$$\delta = \frac{pa_0}{r}$$

$$r = \frac{pa_0}{\delta}$$

But from the diagram resultant amplitude of wave-front will be $S_1 S_2$ (distance between slits i.e. represented by R as maximum amplitude of wave front)

$$\therefore S_1 S_2 = 2 S_1 Q$$

$$\text{As from diagram} \quad S_1 O Q = S_2 O Q = \frac{\delta}{2}$$

$$\text{From } \Delta S_1 O Q \quad \sin \frac{\delta}{2} = \frac{S_1 Q}{OS_1}$$

$$\text{Or} \quad OS_1 \sin \delta/2 = S_1 Q$$

$$\therefore S_1 S_2 = 2 OS_1 \sin \frac{\delta}{2}$$

$$S_1 S_2 = 2r \sin \delta/2$$

$$S_1 S_2 = 2 \frac{pa_0}{\delta} \sin \delta/2$$

$$S_1 S_2 = pa_0 \frac{\sin \delta/2}{\delta/2} = R$$

$$\therefore I = R^2 \quad I = \left[pa_0 \left(\frac{\sin \frac{\delta}{2}}{\frac{\delta}{2}} \right) \right]^2$$

$$I = (pa_0)^2 \left[\left(\frac{\sin \frac{\delta}{2}}{\frac{\delta}{2}} \right) \right]^2$$

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$\text{Here } I_0 = (pa_0)^2 \text{ and } \frac{\delta}{2} = \alpha \quad \text{and} \quad R = pa_0 \left(\frac{\sin \alpha}{\alpha} \right) \quad \therefore \alpha = \frac{\pi}{\lambda} (a \sin \theta)$$

INTENSITY DISTRIBUTION IN SINGLE SLIT DIFFRACTION PATTERN:

1. INTENSITY AND POSITION OF CENTRAL MAXIMA:-

$$\text{At } P_0 \quad \theta = 0^\circ$$

$$\therefore \alpha = \frac{\pi}{\lambda} (a \sin \theta) = 0$$

Then $\frac{\sin \alpha}{\alpha}$ is an indeterminate form

$$\therefore \lim_{\alpha \rightarrow 0} \frac{\sin \alpha}{\alpha} = 1$$

Or $I = I_0 \quad I = (p a_0)^2 \quad (\text{Maximum})$

2. POSITION OF MINIMA:

$$\text{From } I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

We know that intensity will be minimum when $\frac{\sin \alpha}{\alpha} = 0$

But α cannot be zero, because of condition of principal maxima

Since $I = 0$

$$\sin \alpha = 0 \quad \therefore \alpha = \pm n\pi$$

or we can say $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi \dots \dots \dots$

$$\therefore 2\alpha = \frac{2\pi}{\lambda} a \sin \theta$$

$$\alpha = \frac{\pi}{\lambda} a \sin \theta$$

Therefore $\frac{\pi}{\lambda} a \sin \theta = \pm n \pi$

$$a \sin \theta = \pm n \lambda$$

$$\theta = \sin^{-1} \pm \frac{n\lambda}{a}$$

$$\therefore \theta_1 = \sin^{-1} \pm \frac{\lambda}{a}$$

Or $\theta_2 = \sin^{-1} \pm \frac{2\lambda}{a}$

INTENSITY AND POSITION OF SECONDARY MAXIMA:

Since $I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha} \left(I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \right)$$

$$\frac{dI}{d\alpha} = 2 I_0 \left(\frac{\sin \alpha}{\alpha} \right) \left(\frac{\alpha \cos \alpha - \sin \alpha \cdot 1}{\alpha^2} \right)$$

For secondary maxima

$$\frac{dI}{d\alpha} = 0$$

$$\therefore 2I_0 \left(\frac{\sin \alpha}{\alpha} \right) \left(\frac{\alpha \cos \alpha - \sin \alpha \cdot 1}{\alpha^2} \right) = 0$$

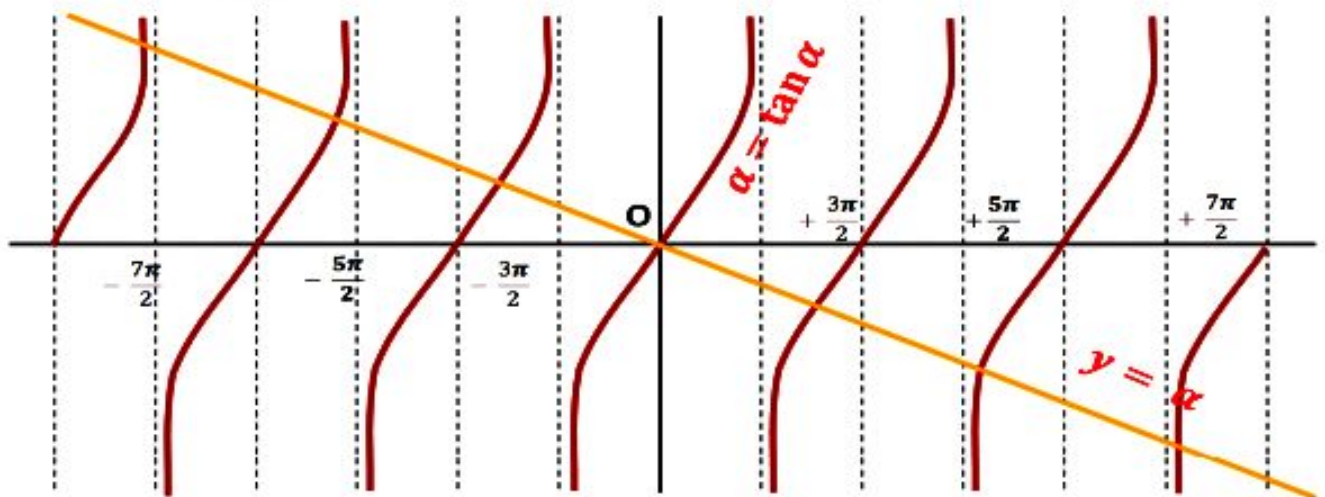
$$I_0 \neq 0 \quad \text{and} \quad \left(\frac{\sin \alpha}{\alpha} \right) \neq 0 \quad \therefore \alpha \neq 0$$

$$\therefore \alpha \cos \alpha - \sin \alpha = 0$$

$$\text{or} \quad \alpha = \tan \alpha \quad \text{and} \quad y = \alpha$$

The intersection point of these two curves will be the position of secondary maxima. The value of α can represent as $\alpha = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2} \dots \dots \dots \pm \frac{(2n+1)\pi}{2}$

$$\text{Since } I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$$



INTENSITY OF 1ST MAXIMA

$$I = I_0 \left(\frac{\sin \frac{3\pi}{2}}{\frac{3\pi}{2}} \right)^2$$

$$I = I_0 \left(\frac{-1}{\frac{3\pi}{2}} \right)^2$$

$$I = I_0 \left(\frac{4}{9\pi^2} \right)$$

$$I = \frac{I_0}{22}$$

Similarly

INTENSITY OF 2ND SECONDARY MAXIMA

$$I = \frac{I_0}{61}$$

Therefore we can say:

$$I_0 : \frac{4I_0}{9\pi^2} : \frac{4I_0}{25\pi^2} : \frac{4I_0}{49\pi^2} \dots \dots \dots$$

$$\text{Or we can say } 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \dots \dots \dots \text{ Or } 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \dots \dots \dots$$

WIDTH OF PRINCIPLE MAXIMA:

The distance between first two minima is called width of principle maxima.

If the lens L2 is very near to slit S_1 and S_2 or the scree is far away from the lens

So from diagram

$$\sin \theta = \frac{x}{f} \quad \text{-----(1)}$$

But $a \sin \theta = n \lambda$

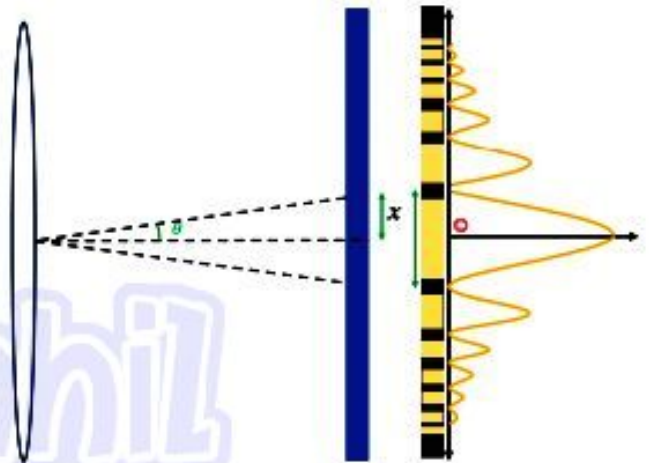
$$\sin \theta = \frac{\lambda}{a} \quad \text{-----(2)}$$

from equation 1 and 2

$$\frac{x}{f} = \frac{\lambda}{a}$$

$$x = \frac{\lambda}{a} \times f$$

$$\omega = 2x = 2 \frac{\lambda}{a} \times f$$



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PLANE TRANSMISSION DIFFRACTION GRATING: -

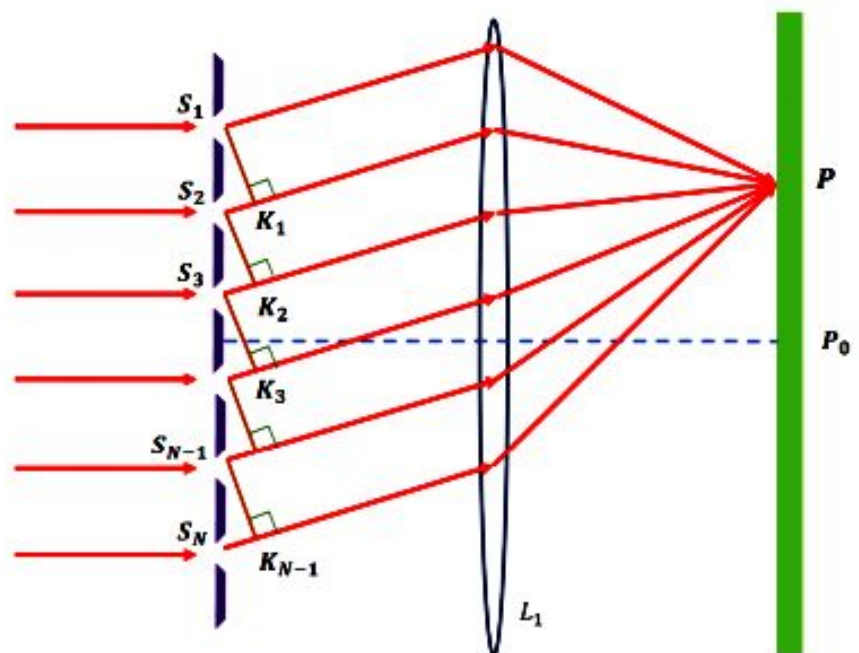
A diffraction grating is a set of multiple identical slits separated by distance comparable to the wavelength of light.

OR

A diffraction grating is an optical device that diffracts light into its constituent wavelength.

CONSTRUCTION:- Gratings are made by ruling a large number of fine, equidistant and parallel lines on an optical plane transparent glass with a fine diamond point. The ruling lines are opaque to light, i.e. it does not allow light to pass through them while the space in between them are transparent.

THEORY: - Let a parallel light beam of monochromatic light of wavelength λ be incident normally on N -parallel slits each of width ' a ' and separated by distance ' b ', such an arrangement providing large number of parallel equidistant slits known as diffraction grating.



The $(a+b)$ is called "**THE DIFFRACTION GRATING ELEMENT**". Let the diffracted light be focused by a lens L_2 on screen XY . The pattern obtained on the screen is the diffraction pattern due to plane transmission diffraction grating.

As we know for single slit resultant amplitude is

$$R = pa_0 \left(\frac{\sin \alpha}{\alpha} \right) \quad \text{and} \quad \alpha = \frac{\pi}{\lambda} a \sin \theta \quad \text{here } \theta = \text{Angle of diffraction}$$

If the total number of slit is N , in result N waves interfere from these N slits and gives interference pattern. (i.e. maxima and minima)

Let $S_1K_1, S_2K_2, S_3K_3, \dots, S_{N-1}K_{N-1}, S_NK_N$ are the perpendicular. Therefore, the path difference can be calculated as from $\Delta S_1S_2K_1$

$$\frac{S_2K_1}{S_1S_2} = \sin \theta$$

$$S_2K_1 = S_1 S_2 \sin \theta$$

$$S_2K_1 = (a+b) \sin \theta \quad \text{-----(1)}$$

$$\text{Similarly } S_3K_2 = (a+b) \sin \theta \quad \text{-----(2)}$$

$$\text{And } S_4K_3 = (a+b) \sin \theta$$

$$\text{Since } \delta = \frac{2\pi}{\lambda} \times \Delta \quad \text{-----(3)}$$

$$\text{Or } \delta = \frac{2\pi}{\lambda} \times (a+b) \sin \theta = 2\beta \text{ (let)} \quad \text{----- (4)}$$

For N waves in the direction θ having a common phase difference 2β and common amplitude is

$$R = p a_0 \left(\frac{\sin \alpha}{\alpha} \right) \quad \text{-----(5)}$$

From diagram let wave front ($\text{arc } S_1S_N$) divided into equal N parts where

$$S_1S_2 = S_2S_3 = S_{N-1}S_N = R$$

In triangle CS_1M

$$S_1S_N = 2 S_1M$$

$$\sin N\beta = \frac{S_1M}{C S_1}$$

$$S_1M = (C S_1) \sin N\beta$$

$$S_1S_N = 2 (S_1M)$$

$$S_1S_N = 2(C S_1) \sin N\beta \quad \text{-----(6)}$$

FROM ΔCNS_1

$$\sin \beta = \frac{S_1N}{C S_1}$$

$$S_1N = C S_1 \sin \beta$$

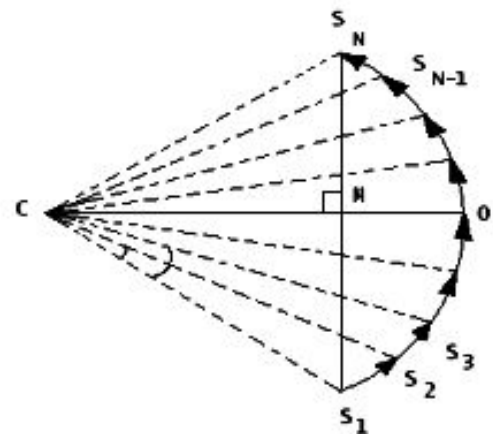
$$S_1S_2 = 2 (S_1N)$$

$$S_1S_2 = 2 (C S_1 \sin \beta) \quad \text{-----(7)}$$

Now from equation (6) and (7)

$$\frac{S_1S_N}{S_1S_2} = \frac{2 C S_1 \sin N\beta}{2 C S_1 \sin \beta}$$

$$S_1S_N = (S_1S_2) \left(\frac{\sin N\beta}{\sin \beta} \right)$$



$$R = (S_1 S_2) = p a_0 \left(\frac{\sin \alpha}{\alpha} \right)$$

$$R_N = S_1 S_N = p a_0 \left(\frac{\sin \alpha}{\alpha} \right) \left(\frac{\sin N\beta}{\sin \beta} \right)$$

$$I = R_N^2$$

$$I = \left[p a_0 \left(\frac{\sin \alpha}{\alpha} \right) \left(\frac{\sin N\beta}{\sin \beta} \right) \right]^2$$

$$I = (p a_0)^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

$$I = (p a_0)^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \frac{\sin^2 N\beta}{\sin^2 \beta}$$

Here $I_0 = p^2 a_0^2$ and $\alpha = \frac{\pi}{\lambda} a \sin \theta$ and $\beta = \frac{\pi}{\lambda} \times (a + b) \sin \theta$

Equation (8) gives the resultant intensity due to 'N' parallel slits transmission grating.

INTENSITY DISTRIBUTION IN DIFFRACTION GRATING: -

INTENSITY AND POSITION OF MAXIMA (PRINCIPLE):

INTENSITY:

$$I = \left[p a_0 \left(\frac{\sin \alpha}{\alpha} \right) \left(\frac{\sin N\beta}{\sin \beta} \right) \right]^2$$

So intensity will be maximum

When $\beta = \pm n\pi$ where $n = 1, 2, 3, \dots$ $\sin N\beta = 0$

therefore $\frac{\sin N\beta}{\sin \beta} = \frac{0}{0}$ (indeterminate form)

$$\begin{aligned} \text{therefore } \lim_{\beta \rightarrow \pm n\pi} \frac{\sin N\beta}{\sin \beta} &= \lim_{\beta \rightarrow \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin \beta)} \\ &= \lim_{\beta \rightarrow \pm n\pi} \frac{N \cos N\beta}{\cos \beta} \end{aligned}$$

$$\lim_{\beta \rightarrow \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \pm N$$

Therefore $I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 N^2$

POSITION OF PRINCIPLE MAXIMA:

Since $\sin \beta = 0$

$$\beta = \pm n \pi \quad \text{therefore} \quad \frac{\pi}{\lambda} (a + b) \sin \theta = n \pi$$

$$\text{Or we can say} \quad (a + b) \sin \theta = \pm n \lambda$$

POSITION OF MINIMA:-

For determining the position of minima

$$\sin N\beta = 0$$

$$N\beta = \pm m \pi$$

$$\beta = \pm \frac{m}{N} \pi$$

$$\text{Therefore} \quad \frac{\pi}{\lambda} (a + b) \sin \theta = \pm \frac{m}{N} \pi \quad \text{so the condition for minima}$$

$$(a + b) \sin \theta = \pm \frac{m \lambda}{N}$$

Where m has all the integral values except $m = 0, 2N, \dots \dots \dots nN$ because these values of m , $(a + b) \sin \theta = \pm n \lambda$ which corresponds to then positions of principal maxima, so condition of minima is

$$(a + b) \sin \theta = \pm \frac{\lambda}{N}, \pm \frac{2\lambda}{N}, \pm \frac{3\lambda}{N}, \dots \dots \dots (N-1) \frac{\lambda}{N},$$

$$(N+1) \frac{\lambda}{N}, (N+2) \frac{\lambda}{N}, (N+3) \frac{\lambda}{N} \dots \dots \dots (2N-1) \frac{\lambda}{N}$$

$$(2N+1) \frac{\lambda}{N}, (2N+2) \frac{\lambda}{N}, (3N+3) \frac{\lambda}{N} \dots \dots \dots (3N-1) \frac{\lambda}{N}$$

Here $m = 0$ gives again a Principal Maxima, $m = 1, 2, 3, \dots \dots \dots (N-1)$ gives minima and then $m = N$ gives the first order principal maximum. Hence there are $(N-1)$ equally spaced minima between two adjacent principal maxima

INTENSITY OF SECONDARY MAXIMA: -

$$\frac{dI}{d\beta} = 0$$

$$\frac{d}{d\beta} \left[I_0 \left(\frac{\sin \alpha}{\alpha} \right) \left(\frac{\sin N\beta}{\sin \beta} \right)^2 \right] = 0$$

$$I_0 \times \left(\frac{\sin \alpha}{\alpha} \right)^2 \times 2 \left(\frac{\sin N\beta}{\sin \beta} \right) \left(\frac{\sin \beta \cos N\beta - \sin N\beta \cos \beta}{\sin^2 \beta} \right) = 0$$

Since $I_0 \times \left(\frac{\sin \alpha}{\alpha}\right)^2 = 0$ gives minima so $I_0 \times \left(\frac{\sin \alpha}{\alpha}\right)^2 \neq 0$

And $2 \left(\frac{\sin N\beta}{\sin \beta}\right) = 0$ gives PRINCIPAL MAXIMA so $2 \left(\frac{\sin N\beta}{\sin \beta}\right) \neq 0$

therefore only $\sin \beta \cdot \cos N\beta - \sin N\beta \cos \beta = 0$

$$N \tan \beta = \tan N\beta$$

$$\text{Or } \tan N\beta = N \tan \beta$$

Drawing right angle triangle for this equation

From the geometry

$$\sin N\beta = \frac{N \tan \beta}{\sqrt{1 + N^2 \tan^2 \beta}}$$

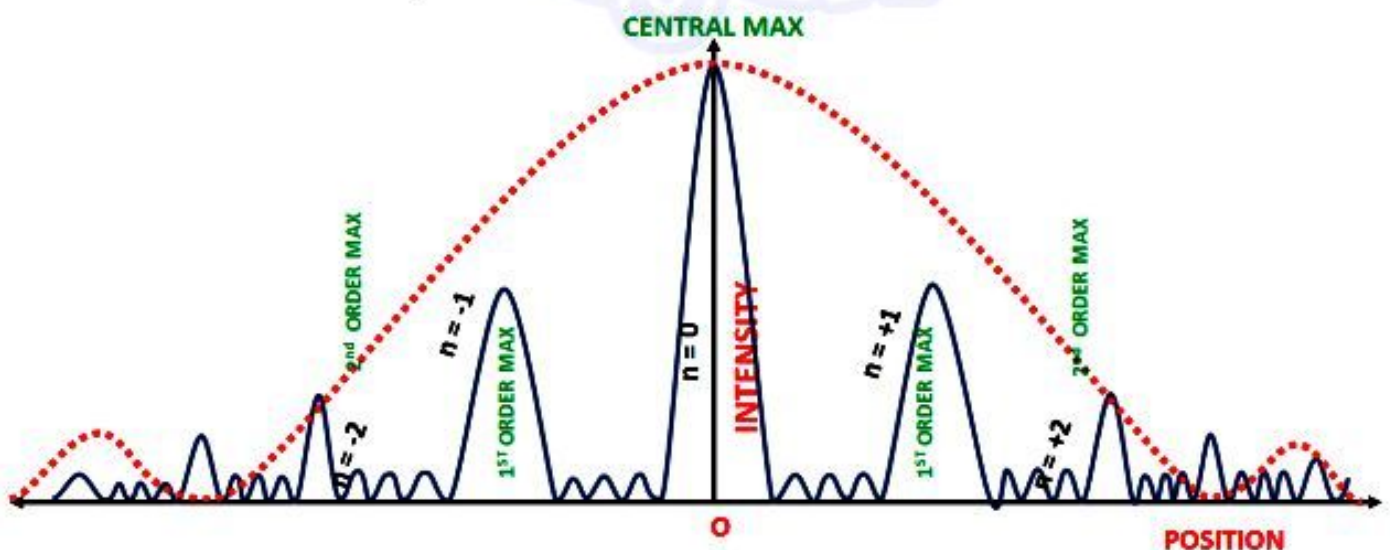
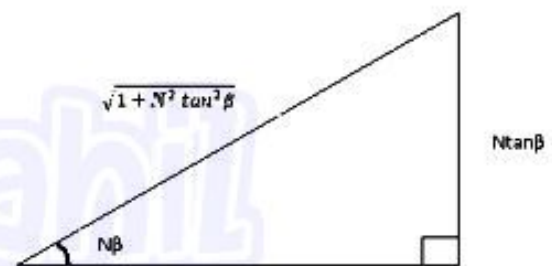
Squaring both side

$$(\sin N\beta)^2 = \frac{N^2 \tan^2 \beta}{1 + N^2 \tan^2 \beta}$$

$$\sin^2 N\beta = \frac{N^2 \frac{\sin^2 \beta}{\cos^2 \beta}}{1 + N^2 \frac{\sin^2 \beta}{\cos^2 \beta}}$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{1}{\sin^2 \beta} \left(\frac{N^2 \frac{\sin^2 \beta}{\cos^2 \beta}}{1 + N^2 \frac{\sin^2 \beta}{\cos^2 \beta}} \right)$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2}{\cos^2 \beta (1 + N^2 \frac{\sin^2 \beta}{\cos^2 \beta})}$$



$$= \frac{N^2}{\cos^2 \beta + N^2 \sin^2 \beta}$$

$$= \frac{N^2}{(1 - \sin^2 \beta) + N^2 \sin^2 \beta}$$

$$= \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{N^2}{1 + (N^2 - 1) \sin^2 \beta} \right)$$

DETERMINATION OF WAVELENGTH USING PLANE TRANSMISSION GRATING: -

The wavelength of light spectrum can be measure using PTG, if the angle of diffraction is θ for the spectral line of wavelength λ in the nth order of spectrum, then

$$(a + b) \sin \theta_n = n\lambda \quad \text{----- (1)}$$

If PTG have 15000 lines per inch so

$$(a + b) = \frac{2.54}{15000} = 1.69 \times 10^{-6} \text{ m.}$$

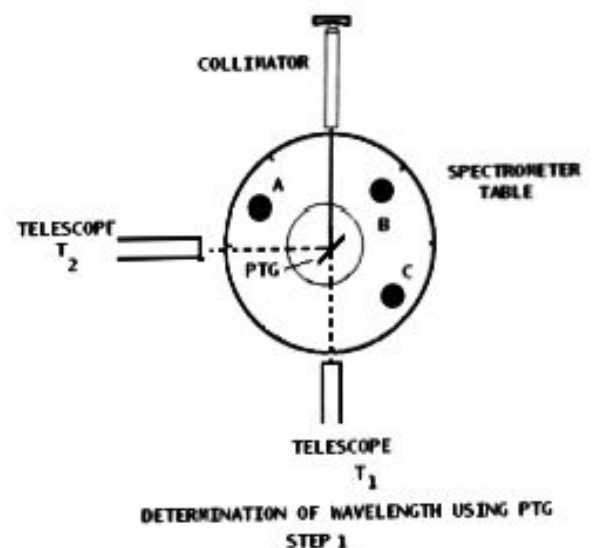
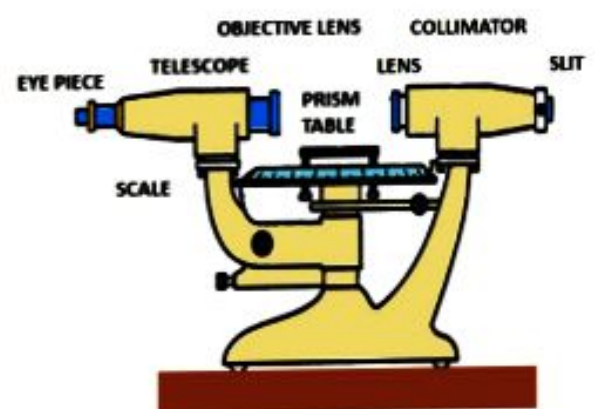
EXPERIMENT: - Before performing the experiment the following adjustment are made:

- Eyepiece of telescope is focus on the cross wire
- The collimator is adjusted to produce parallel rays and the telescope to focus the parallel rays.

The grating is adjusted on prism table normal to the collimator for this purpose the slit of collimator is illuminated by the source of light.

Now the position of telescope is adjusted such a way that the image of the slit is focused on the vertical cross wire of the telescope. In this position the collimator and the telescope are in the same line (T_1 Position). The position of T_1 is noted on circular scale.

Now it is turned at 90° (T_2 position) and clamped (tightened). The given PTG is mounted at the centre of prism table such that the PTZ is perpendicular to the table.



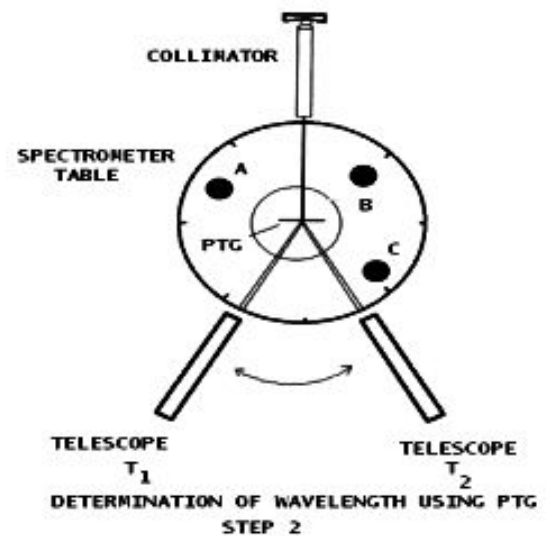
The prism table is now rotated so that the image of the slit reflected from the grating surface lies at the intersection of the cross wire. In this position the grating is at 45° to the incident light. The prism table is suitably rotated through 45° in such a way that grating is exactly normal to the incident light. The prism table is clamped in the position.

When the source emits light of different wavelength the beam get dispersed by the grating and in each order a spectrum of each wavelength is observed. The telescope is now turned to get the 1st order spectrum. The readings of the vernier are recorded.

The readings of the same vernier give twice the angle of diffraction for that line in the first order by putting the value of θ in

$$\lambda = \frac{(a+b) \sin \theta_n}{n}$$

Or
$$\lambda = \frac{(1.69 \times 10^{-6}) \sin \theta_n}{n}$$



TO DETERMINE THE ABSENT SPECTRA :

If the angle of diffraction θ is such that minima due to the diffraction component in the intensity distribution fall at the same position where principal maxima due to interference component is situated then that order of principal maxima will be absent in the spectrum such spectra are called absent spectra.

The condition of minima in diffraction pattern is given by

$$a \sin \theta = \pm m \lambda \quad \text{----- (1)}$$

If the n^{th} order principal maximum lies in this direction then

$$(a + b) \sin \theta = \pm n \lambda \quad \text{----- (2)}$$

From (1) and (2)

$$\frac{(a+b)}{a} = \frac{n}{m}$$

So order of Absent spectra

$$n = \left(\frac{a+b}{a} \right) m$$

If $b = a$ then $n = 2m$

Since $m = 1, 2, 3, \dots$ then $n = 2, 4, 6, \dots$

If $b = 2a$ then $n = 3m$

Since $m = 1, 2, 3, \dots$ then $n = 3, 6, 9, \dots$

MAXIMUM NUMBER OF ORDER OF SPECTRUM OBSERVED:

Principal maxima in grating spectrum is given by:

$$(a + b) \sin \theta = \pm n \lambda \quad \text{----- (2)}$$

Order of Principal maxima

$$n = \pm \frac{(a+b) \sin \theta}{\lambda}$$

As maximum possible angle of diffraction is $\pm 90^\circ$

$$n_{\max} = \pm \frac{(a+b)}{\lambda}$$

If $(a + b) < \lambda$ then $n_{\max} < 1$ Only zero order max will be observed.

If $(a + b) < 2\lambda$ then $n_{\max} = \frac{a+b}{\lambda} < 2$ that is only first order Principal max. will be observed the second and higher orders will be absent and so on.

WIDTH OF PRINCIPAL MAXIMA:

The condition of n^{th} order Principal maxima of the wavelength λ in the direction of angle θ is given by:

$$(a + b) \sin \theta = \pm n \lambda \quad \text{----- (1)}$$

Now let the nearest minima on both side of principal maxima are in the direction of $(\theta - d\theta)$ and $(\theta + d\theta)$ then $2d\theta$ will be angular width of Principal maxima.

For the position of minima

$$N(a + b) \sin(\theta \pm d\theta) = p\lambda \quad \text{----- (2)}$$

Where p is a positive integer except 0, $N, 2N, \dots$ Where N is total number of slits. For n^{th} order maxima will be

$$N(a + b) \sin \theta = n N \lambda$$

$\frac{p}{N} = n$ or $p = n N$ will give the position of Principal Max. So for the minima to the n^{th} order Principal Max.

$$p = n N \pm 1$$

$$\therefore (a + b) \sin(\theta \pm d\theta) = (n N \pm 1) \frac{\lambda}{N}$$

$$(a + b) \sin(\theta \pm d\theta) = n \lambda \pm \frac{\lambda}{N}$$

From equation (1) and (3)

$$\frac{\sin(\theta \pm d\theta)}{\sin \theta} = \left(1 \pm \frac{1}{nN}\right)$$

$$\frac{\sin \theta \cos d\theta \pm \cos \theta \sin d\theta}{\sin \theta} = 1 \pm \frac{1}{nN}$$

Since $d\theta$ is small, so $\cos d\theta \approx 1$ and $\sin d\theta \approx d\theta$

$$1 \pm \cot \theta \, d\theta = 1 \pm \frac{1}{nN}$$

$$d\theta = \frac{\tan \theta}{nN} = \frac{\sin \theta}{nN \cos \theta}$$

But $\frac{\sin \theta}{n} = \frac{\lambda}{a+b}$

$$d\theta = \frac{\lambda}{(a+b)N \cos \theta}$$

Therefore, angular width

$$2 \, d\theta = \frac{2 \, \lambda}{(a+b) N \cos \theta}$$

RESOLVING POWER:- When two objects or image are very close together, they may appear as one, and it may not be possible for eyes to see them as separate. The method of separating out such close objects or images is called RESOLUTION.

The ability of an optical instrument or eyes to form separate images of such two objects (very close) is called its resolving power.

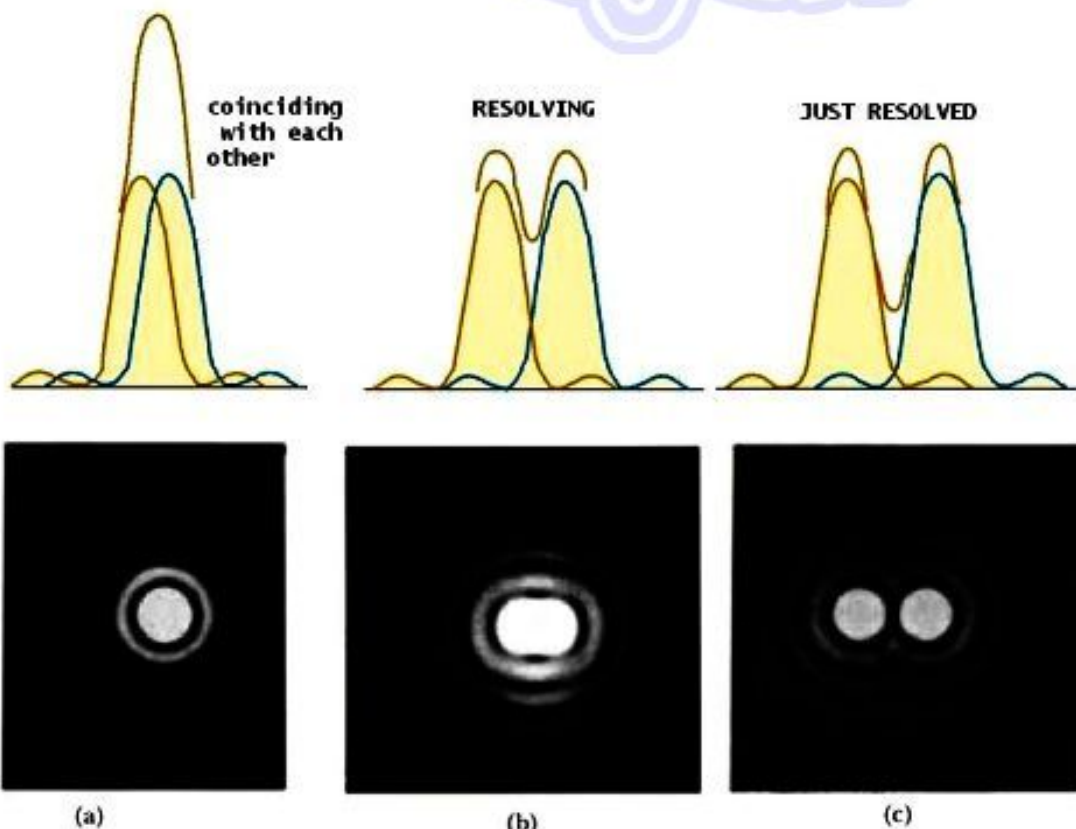
$$\text{RESOLVING POWER} = \frac{1}{\text{RESOLUTION}}$$

Resolving power of eyes is about 1° or we can say

$$\theta = \frac{a}{d} = 2 \times \frac{10^{-3}}{5} = 4 \times 10^{-4} \text{ rad.}$$

RAYLEIGH'S CRITERION FOR RESOLUTION :-

According to Rayleigh, two close point sources or images of nearly equal intensity are said to be just resolved by an optical instrument, when the central maxima of the diffraction pattern of one source coincides with the first minimum of the diffraction pattern of the other. In case of just resolution the distance or angular separation between the two sources is called resolution limit.



QUANTUM MECHANICS

- Compton Effect and quantum nature of light
- Derivation of time dependent and time independent Schrödinger's Wave Equation.
- Physical interpretation of wave function and its properties, Boundary Conditions, Particle in one dimensional box.

QUANTUM MECHANICS:

INTRODUCTION TO QUANTUM MECHANICS

Quantum mechanics is a physical science dealing with the behavior of matter and energy on the scale of atoms and subatomic particles / waves.

It also forms the basis for the contemporary understanding of how very large objects such as stars and galaxies, and cosmological events such as the Big Bang, can be analyzed and explained.

Quantum mechanics is the foundation of several related disciplines including nanotechnology, condensed matter physics, quantum chemistry, structural biology, particle physics, and electronics.

In other ways we can say that the exchange of energy between the radiation and the matter takes place in discrete set of values. In view of the application of quantum theory, it is necessary to be aware of the photon.

PHOTON : MASS, ENERGY AND MOMENTUM

Photon is an elementary particle, that is massless and has no charge. It is a bundle of energy or packet of energy emitted by a source of radiation. It moves with velocity of light. It can carry energy and momentum. We know the relation between velocity of light and mass of particle is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Where m is the relativistic mass of the particle and m_0 is its rest mass. Since the photon is moving with the velocity of the light, we substitute $v=c$ in such equation, mass of photon becomes $m = \infty$, i.e. not possible. But if we take $m_0 = \text{zero}$ and $v=c$ then this yields $m = 0/0$ this is indeterminate quantity. It means if we take rest mass of the photon to be zero, this value should not particularly disturb us due the fact that the photons are never at rest and always keep moving with the velocity of light.

The energy of a photon is given below as

$$E = h\nu \quad \text{And} \quad E = mc^2$$

$$\text{so} \quad mc^2 = h\nu \quad \text{Or} \quad m = \frac{h\nu}{c^2}$$

$$\text{Since the energy relation} \quad E^2 = p^2c^2 + m_0^2c^4$$

Since $m_0 = 0$ and $E = pc$ and the momentum of the photon is given by

$$p = \frac{E}{c} = \frac{mc^2}{c} = mc$$

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

Thus, if a photon of frequency ν is to be treated as a particle, then the characteristics of the photon are given as:

$$m_0 = 0, E = h\nu, m = \frac{h\nu}{c^2} \text{ and } p = \frac{h\nu}{c}$$

WAVE PARTICLE DUALITY:

The phenomena of interference, diffraction and polarization can be explained on the basis of wave theory of light. However, the wave nature of light fails to explain the phenomena of Compton effect, photoelectric effect, the continuous X-ray spectrum and the blackbody radiation. In the light of these facts we can say, is the particle nature of electromagnetic radiation. These entire phenomena can be explained on the basis of quantum hypothesis, according to which electromagnetic radiation is propagated in small packets or bundles. These packets are called photons. It means that light or electromagnetic radiation exhibits wave and particle properties both. Hence, light or electromagnetic radiation has dual nature, i.e. it behaves like a particle as well as a wave. This dual characteristic property of radiation is called dual nature of light or wave particle duality.

COMPTON SCATTERING:

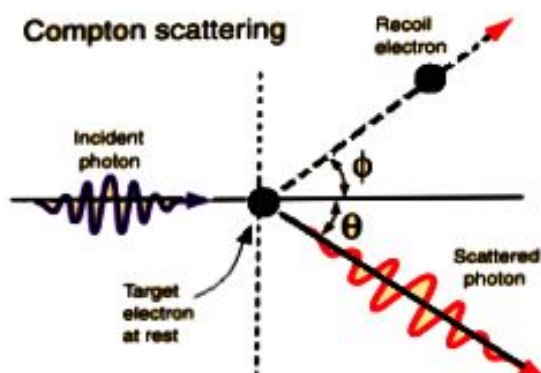
When high frequency monochromatic (X-RAY) beam is struck on material then there is scattering of waves of two types out of material which, one has less frequency than the incident radiation and other has same frequency of the incident radiation. This phenomenon is known as COMPTON SCATTERING.

COMPTON EFFECT BY QUANTUM THEORY:

According to this theory the radiation is quantized i.e., it is emitted in the form of bundles or packets of energy called photon. In Compton Effect due to an elastic collision between photon and electron, photon transfer some energy to electron i.e., it losses energy. The scattered photon will have smaller energy $h\nu'$ and consequently a lower frequency than that of incident photon.

THEORY:

Let a photon of energy $h\nu$ collides with a free electron i.e. in rest. After collision it gains some kinetic energy and starts moving in or at an angle of ϕ and the scattered photon start moving at an angle of θ with energy $h\nu'$ and momentum $\frac{h\nu'}{c}$.



ENERGY	PHOTON	ELECTRON	
BEFORE COLLISION	$h\nu$	$m_0 c^2$	
AFTER COLLISION	$h\nu'$	mc^2	
MOMENTUM	PHOTON	ELECTRON	
BEFORE COLLISION	$h\nu/c$	0	X AXIS
AFTER COLLISION	$\frac{h\nu' \cos \theta}{c}$	$mv \cos \phi$	
MOMENTUM	PHOTON	ELECTRON	
BEFORE COLLISION	0	0	Y AXIS

AFTER COLLISION	$\frac{hv' \sin \theta}{c}$	$mv \sin \phi$	
-----------------	-----------------------------	----------------	--

According to conservation of energy

TOTAL ENERGY BEFORE COLLISION = TOTAL ENERGY AFTER COLLISION

$$hv + m_0c^2 = hv' + mc^2 \quad \text{-----(1)}$$

According to conservation of momentum along the direction of incident photon (X axis).

$$\frac{hv}{c} + 0 = \frac{hv' \cos \theta}{c} + mv \cos \phi \quad \text{-----(2)}$$

According to conservation of momentum perpendicular to the direction of incident photon (Y axis).

$$0 + 0 = \frac{hv' \sin \theta}{c} - mv \sin \phi \quad \text{-----(3)}$$

Rewriting the equation (2) and (3)

$$hv - hv' \cos \theta = mvc \cos \phi \quad \text{-----(4)}$$

$$hv' \sin \theta = mvc \sin \phi \quad \text{-----(5)}$$

squaring and adding equation (4) and (5)

$$\begin{aligned} m^2v^2c^2 (\cos^2 \phi + \sin^2 \phi) &= (hv - hv' \cos \theta)^2 + (hv' \sin \theta)^2 \\ m^2v^2c^2 &= h^2v^2 + h^2v'^2 \cos^2 \theta - 2h^2vv' \cos \theta + h^2v'^2 \sin^2 \theta \\ m^2v^2c^2 &= h^2v^2 + h^2v'^2 (\cos^2 \theta + \sin^2 \theta) - 2h^2vv' \cos \theta \\ m^2v^2c^2 &= h^2v^2 + h^2v'^2 - 2h^2vv' \cos \theta \\ m^2v^2c^2 &= h^2(v^2 + v'^2 - 2vv' \cos \theta) \quad \text{-----(6)} \end{aligned}$$

$$\text{now from equation (1)} \quad hv + m_0c^2 = hv' + mc^2$$

$$h(v - v') + m_0c^2 = mc^2 \quad \text{-----(7)}$$

squaring this equation (7)

$$\begin{aligned} (h(v - v') + m_0c^2)^2 &= (mc^2)^2 \\ m^2c^4 &= h^2(v - v')^2 + m_0^2c^4 + 2h(v - v') m_0c^2 \\ m^2c^4 &= h^2(v^2 + v'^2 - 2vv') + m_0^2c^4 + 2h(v - v') m_0c^2 \\ m^2c^4 &= h^2v^2 + h^2v'^2 - 2h^2vv' + m_0^2c^4 + 2h(v - v') m_0c^2 \quad \text{-----(8)} \end{aligned}$$

Now subtracting equation (6) and (8)

$$m^2 c^4 - m^2 v^2 c^2 = h^2 v^2 + h^2 v'^2 - 2h^2 v v' + m_0^2 c^4 + 2h(v - v') m_0 c^2 - h^2 v^2 - h^2 v'^2 + 2h^2 v v' \cos \theta$$

$$m^2 c^4 \left(1 - \frac{v^2}{c^2}\right) = -2h^2 v v' (1 - \cos \theta) + m_0^2 c^4 + 2h(v - v') m_0 c^2 \quad \text{-----(9)}$$

as we know that $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\left(\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\right)^2 c^4 \left(1 - \frac{v^2}{c^2}\right) = -2h^2 v v' (1 - \cos \theta) + m_0^2 c^4 + 2h(v - v') m_0 c^2$$

$$\left(\frac{m_0^2}{1 - \frac{v^2}{c^2}}\right) c^4 \left(1 - \frac{v^2}{c^2}\right) = -2h^2 v v' (1 - \cos \theta) + m_0^2 c^4 + 2h(v - v') m_0 c^2$$

$$m_0^2 c^4 = -2h^2 v v' (1 - \cos \theta) + m_0^2 c^4 + 2h(v - v') m_0 c^2$$

$$0 = -2h^2 v v' (1 - \cos \theta) + 2h(v - v') m_0 c^2$$

$$2h^2 v v' (1 - \cos \theta) = 2h(v - v') m_0 c^2$$

$$(v - v') = \frac{h v v' (1 - \cos \theta)}{m_0 c^2}$$

$$\frac{1}{v'} - \frac{1}{v} = \frac{h(1 - \cos \theta)}{m_0 c^2}$$

$$\frac{c}{v'} - \frac{c}{v} = \frac{h(1 - \cos \theta)}{m_0 c}$$

$$\lambda' - \lambda = \frac{h(1 - \cos \theta)}{m_0 c}$$

$$\lambda' - \lambda = \frac{2h \sin^2 \theta/2}{m_0 c}$$

This difference of wavelength is called as Compton wavelength shift.

COMPTON WAVELENGTH:

Since from Compton shift $\lambda' - \lambda = \frac{2h \sin^2 \theta/2}{m_0 c}$

let $\frac{h}{m_0 c} = \lambda_c$ here $h = 6.625 \times 10^{-34} \text{ Js}$ (Planck's constant)

$$m_0 = 9.11 \times 10^{-31} \text{ Kg.}$$

$$c = 3 \times 10^8 \text{ m./sec}$$

therefore $\lambda_c = 0.0242 \text{ \AA}$

and $\lambda' - \lambda = 0.0242 (1 - \cos \theta)$

If	$\theta = \frac{\pi}{2}$	$\cos \theta = 0$	Therefore	$\Delta\lambda = \lambda' - \lambda = \lambda_c$
If	$\theta = \pi$	$\cos \theta = -1$	Therefore	$\Delta\lambda = 2 \lambda_c = 0.0484$

DIRECTION OF RECOILED ELECTRON:

Since the conservation of momentum in X- axis and Y- axis

$$hv - hv' \cos \theta = mvc \cos \phi \quad \text{-----(1)} \quad \text{(on X axis)}$$

$$hv' \sin \theta = mvc \sin \phi \quad \text{-----(2)} \quad \text{(on Y axis)}$$

from equation (1) and (2)

$$\frac{mvc \sin \phi}{mvc \cos \phi} = \frac{hv' \sin \theta}{hv - hv' \cos \theta}$$

$$\tan \phi = \frac{v' \sin \theta}{v - v' \cos \theta} \quad \text{-----(3)}$$

since Compton shift is $\frac{1}{v'} - \frac{1}{v} = \frac{h(1 - \cos \theta)}{m_0 c^2}$ Multiplying with v

$$\frac{v}{v'} - 1 = \frac{hv(1 - \cos \theta)}{m_0 c^2}$$

$$\frac{v}{v'} = 1 + \frac{hv(1 - \cos \theta)}{m_0 c^2} \quad \text{-----(4)}$$

Let $\frac{hv}{m_0 c^2} = \alpha$

Therefore $\frac{v}{v'} = 1 + \alpha (1 - \cos \theta)$ -----(5)

$$\tan \phi = \frac{v' \sin \theta}{v - v' \cos \theta} = \frac{\sin \theta}{\frac{v}{v'} - \cos \theta}$$

Putting value of $\frac{v}{v'}$ from equation (5)

$$\tan \phi = \frac{\sin \theta}{[1 + \alpha (1 - \cos \theta)] - \cos \theta}$$

$$\tan \phi = \frac{\sin \theta}{1 + \alpha - \alpha \cos \theta - \cos \theta}$$

$$\tan \phi = \frac{\sin \theta}{(1 + \alpha)(1 - \cos \theta)} \quad \text{since } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\tan \phi = \frac{2 \sin \theta / 2 \cos \theta / 2}{(1 + \alpha) 2 \sin^2 \theta / 2}$$

$$\tan \phi = \frac{\cot \theta / 2}{1 + \alpha} \quad \text{here } \alpha = \frac{hv}{m_0 c^2} \quad \tan \phi = \frac{\cot \theta / 2}{1 + \frac{hv}{m_0 c^2}}$$

KINETIC ENERGY OF RECOILED ELECTRON:

According to energy conservation:

Energy lost by photon = Energy gained by electron

$$hv - hv' = \text{KINETIC ENERGY}$$

$$K.E. = h(\nu - \nu')$$

$$K.E. = h\nu \left(1 - \frac{\nu'}{\nu}\right)$$

From equation 5 $\frac{\nu}{\nu'} = 1 + \alpha (1 - \cos \theta)$

$$K.E. = h\nu \left(1 - \frac{1}{(1 + \alpha (1 - \cos \theta))}\right)$$

$$K.E. = h\nu \alpha \frac{(1 - \cos \theta)}{1 + \alpha (1 - \cos \theta)} \quad \text{since } 1 - \cos \theta = \sin^2 \theta / 2$$

Therefore $K.E. = h\nu \alpha \frac{2 \sin^2 \theta / 2}{1 + 2 \sin^2 \theta / 2}$

(i) When $\theta = \pi$ $K.E. = h\nu \frac{2\alpha}{1+2\alpha}$ (MAXIMUM)

(ii) When $\theta = \pi/2$ $K.E. = h\nu \frac{\alpha}{1+\alpha}$

(iii) When $\theta = 0$ $K.E. = 0$ (MINIMUM)

FREQUENCY AND ENERGY OF THE SCATTERED PHOTON: The frequency of scattered photon is given by ν' . Since we know

$$\frac{\nu}{\nu'} = 1 + \alpha (1 - \cos \theta) \quad \text{and} \quad \alpha = \frac{h\nu}{m_0 c^2}$$

$$\nu' = \frac{\nu}{1 + \alpha (1 - \cos \theta)}$$

SO, THE ENERGY OF SCATTERED PHOTON:

$$h\nu' = \frac{h\nu}{1 + \alpha (1 - \cos \theta)}$$

(i) When $\theta = \pi$ if photon scattered backward then

$$h\nu' = \frac{h\nu}{1 + 2\alpha}$$

(ii) When $\theta = \pi/2$ When photon is scattered perpendicularly

$$h\nu' = \frac{h\nu}{1 + \alpha}$$

(iii) When $\theta = 0$ There is no scattering then

$$h\nu' = h\nu$$

WAVE FUNCTION:

Waves in general are associated with quantities that vary periodically. In case of matter waves, the quantity that varies periodically is called wave function.

The wave function is associated with a moving particle at particular point (x, y, z) in space time 't' gives the complete description of motion and behavior of particle under the potential applied.

It is denoted by Ψ (SCI)

Wave function has no physical signification but it gives the probability of finding the particle at any point or anywhere in the space at time 't'.

POSTULATES OF QUANTUM MECHANICS:

1. Wave function $\Psi(x, y, z, t)$ is a quantum mechanical state which describes a physical system completely

2. The intensity of a wave motion is proportional to the square of amplitude, i.e. we can say is

$$|\psi(x, y, z, t)|^2$$

3. The wave function can be expressed by a complex function so its conjugate will ψ^*

$$\text{Then } |\psi(x, y, z, t)|^2 = \psi^*(x, t) \times \psi(x, t)$$

$$\text{If } \psi(x, t) \text{ is real then } \psi^* = \psi$$

4. The probability of finding a particle in an interval dx of x at time t is

$$P = |\psi(x, y, z, t)|^2 dx$$

Similarly, probability of finding a particle in a volume element $dv = dx dy dz$ about at any point $\mathbf{r}$ at a time 't' is given by

$$P(\mathbf{r}) dv = |\psi(x, y, z, t)|^2 dV$$

$$P(\mathbf{r}) dv = \psi^*(x, y, z, t) \cdot \psi(x, y, z, t) dV$$

Here $P(\mathbf{r})$ is defined as the probability density or probability per unit volume.

$$\text{So total probability} = \int_{-\infty}^{\infty} |\psi(x, y, z, t)|^2 dV$$

$$\text{If } \int_{-\infty}^{\infty} |\psi(x, y, z, t)|^2 dv = 0 \quad \text{Then particle does not exist}$$



If $\int_{-\infty}^{\infty} |\Psi(x, y, z, t)|^2 dv = 1$ Particle exist somewhere in space at all time

NORMALIZATION CONDITION:

Since the probability is equal to one then

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx dy dz = \int_{-\infty}^{\infty} \Psi dx dy dz = 1$$

This is known as normalization condition.

ORTHOGONAL WAVE FUNCTION:

When Two different wave function are multiplied and integrate with respect to all space, results zero then this condition is known as orthogonal condition and this kind of functions are called MUTUAL ORTHOGONAL.

$$\int_{\text{all space}} \psi_i \psi_j^* dV = 0$$

$$\int_{\text{all space}} \psi_j \psi_i^* dV = 0$$

Here $i \neq j$

AVERAGE VALUE:

$$\langle q \rangle = \frac{\int_{-\infty}^{\infty} \Psi^* Q \Psi dv}{\int_{-\infty}^{\infty} \Psi^* \Psi dv}$$

8. A wave function Ψ must satisfy the condition that the wave function Ψ and its first derivatives i.e. Ψ and $\frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial z}$ must be finite, continuous, single valued at everywhere and zero at larger distances.

SCHRODINGER'S EQUATION:

Schrodinger's Equation gives the description of dynamics of quantum Mechanics.

In classical mechanics the state of motion of a particle is specified by giving the particle's position and velocity.

In Quantum Mechanics the state of motion of a particle is specified by giving the wave function.

The classical equation of motion is $f=ma$. In Quantum Mechanics the equation of motion is SCRODINGER'S TIME DEPENDENT EQUATION. Which determine the wave function at any time 't'

TIME DEPENDENT SCHRODINGER'S EQUATION:-

Let a particle is moving along X- Axis so there must be a wave with moving particle. The wave function Ψ is given by:

$$\psi(x, t) = A e^{i(kx - \omega t)} \quad \text{-----(1)}$$

Here $k = 2\pi/\lambda$ (Wave Number)

$$\omega = \text{Angular frequency} = 2\pi\nu \quad \text{-----(2)}$$

$$\therefore E = h\nu \quad \text{-----(3)}$$

$$h = \frac{h}{2\pi} \quad \text{or} \quad h = \hbar 2\pi \quad \text{-----(4)}$$

$$\therefore E = 2\pi \hbar \nu \quad \text{or} \quad E = \hbar \omega \quad \text{-----(5)}$$

$$\omega = E/\hbar \quad \text{-----(6)}$$

and momentum $p = h/\lambda$ putting value of h from equation 4

$$p = \hbar 2\pi/\lambda \quad \text{since} \quad k = 2\pi/\lambda$$

$$p = \hbar k \quad \text{or} \quad k = p/\hbar \quad \text{-----(7)}$$

so we get $\omega = \frac{E}{\hbar}$ and $k = \frac{p}{\hbar}$ putting these values of ω and k in equation (1)

$$\psi(x, t) = A e^{i(kx - \omega t)} \quad \text{-----(1)}$$

$$\psi(x, t) = A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)}$$

$$\psi(x, t) = A e^{\frac{i}{\hbar}(px - Et)} \quad \text{-----(8)}$$

This equation is called as wave equation for a free particle.

ENERGY OPERATOR:

Partially differentiating equation (8) with respect to 't' gives:

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} A e^{-\frac{i}{\hbar}(px - Et)}$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi(x, t)$$

$$E \psi(x, t) = \frac{\hbar}{-i} \frac{\partial \psi}{\partial t}$$

$$E \psi(x, t) = i \hbar \frac{\partial \psi}{\partial t} \quad \quad \quad E \equiv i \hbar \frac{\partial}{\partial t} \quad \quad \quad \text{This is known as Energy operator.}$$

Similarly, we can derive momentum operator by differentiating equation (6) with respect to 'x'

$$\frac{\partial \psi}{\partial x} = \frac{-ip}{\hbar} A e^{-\frac{i}{\hbar}(px - Et)}$$

$$\frac{\partial \psi}{\partial x} = \frac{-ip}{\hbar} \psi(x, t)$$

$$p \psi(x, t) = -i \hbar \frac{\partial \psi}{\partial x}$$

$$\mathbf{p} \equiv i \hbar \frac{\partial}{\partial x} \quad \text{This is known as Momentum Operator.}$$

Since we know total energy $E = K.E. + P.E.$

$$\text{Since } K.E. = \frac{1}{2} m v^2 = \frac{p^2}{2m} \quad \text{therefore} \quad E = \frac{p^2}{2m} + V$$

$$\text{Or we can say } E \psi = \frac{p^2}{2m} \psi + V \psi$$

$$\text{or } i \hbar \frac{\partial \psi}{\partial t} = \frac{(-i \hbar \frac{\partial \psi}{\partial x})^2}{2m} + V \psi$$

$$\text{or } i \hbar \frac{\partial \psi}{\partial t} = - \left(- \hbar \frac{\partial}{\partial x} \right)^2 \frac{\psi}{2m} + V \psi$$

$$i \hbar \frac{\partial \psi}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi$$

or we can rewrite

$$i \hbar \frac{\partial \psi(x, t)}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + V \psi(x, t) \quad \text{-----(7)}$$

This equation is known as SCHRÖDINGER'S TIME DEPENDENT ONE DIMENSIONAL WAVE EQUATION.

For 3D form

$$i \hbar \frac{\partial \psi(r, t)}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \psi(r, t)}{\partial x^2} + V \psi(r, t)$$

$$i \hbar \frac{\partial \psi(r, t)}{\partial t} = - \frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(r, t) + V(r, t) \psi(r, t) \quad \text{----- (8)}$$

$$i \hbar \frac{\partial \psi(r, t)}{\partial t} = - \frac{\hbar^2}{2m} \nabla^2 \psi(r, t) + V(r, t) \psi(r, t) \quad \text{----- (9)}$$

$$\text{Here } \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$

So rewriting the equation (9) as

$$i \hbar \frac{\partial \psi(r, t)}{\partial t} = \left[- \frac{\hbar^2}{2m} \nabla^2 + V(r, t) \right] \psi(r, t)$$

$$i \hbar \frac{\partial \psi(r, t)}{\partial t} = H \psi(r, t) \quad \text{-----(10)}$$

$$-\frac{\hbar^2}{2m} \nabla^2 + V(r, t) \equiv H \quad \text{(HAMILTONIAN OPERATOR)} \quad \text{-----(11)}$$

TIME INDEPENDENT SCHRODINGER'S EQUATION:

Since we know wave function is

$$\psi(x, t) = A e^{\frac{i}{\hbar}(p x - Et)}$$

Separating variable, we can write

$$\psi(x, t) = A e^{\frac{i}{\hbar}(p x)} \cdot e^{\frac{i}{\hbar}(-Et)}$$

$$\psi(x, t) = \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)} \quad \text{-----(1)}$$

Partially differentiating with respect to 't' yields

$$\frac{\partial}{\partial t} \psi(x, t) = \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)} \cdot \left(-\frac{iE}{\hbar}\right) \quad \text{-----(2)}$$

Partially differentiating with respect to 'x' yields

$$\frac{\partial}{\partial x} \psi(x, t) = \frac{\partial}{\partial x} \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)}$$

Again partially differentiating with respect to 'x' yields

$$\frac{\partial^2}{\partial x^2} \psi(x, t) = \frac{\partial^2}{\partial x^2} \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)} \quad \text{-----(3)}$$

Since we know time dependent SCHRODINGER'S EQUATION is:

$$i \hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + V \psi(x, t)$$

Replacing the values from equation (1), (2) and (3) and also replacing $V(x, t)$ to $V(x)$.

$$i \hbar \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)} \cdot \left(-\frac{iE}{\hbar}\right) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)} + V(x) \cdot \psi(x) \cdot e^{-\frac{i}{\hbar}(Et)} \quad \text{-----(4)}$$

$$-(-1) E \psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x) \cdot \psi(x)$$

$$[E - V(x)] \psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x)$$

$$\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + [E - V(x)] \psi(x) = 0 \quad \text{-----(5)}$$

$$\frac{\partial^2}{\partial x^2} \psi(x) + \frac{2m}{\hbar^2} [E - V(x)] \psi(x) = 0 \quad \text{-----(6)}$$

This equation is known as TIME INDEPENDENT 1-D SCHRODINGER'S EQUATION.

Similarly we can write TIME INDEPENDENT 3-D SCHRODINGER'S WAVE EQUATION.

FREE PARTICLE IN A ONE DIMENSIONAL BOX:

Let us suppose that a particle of m mass is moving along X - Axis in one dimensional box. The particle is free to move between $x = 0$ and $x = a$ (between the walls of box). Since the potential at the walls is infinite so particle cannot jump or penetrate the walls. So, particle does not exist outside of walls of box. So the wave function outside the box.

$$\psi(x) = 0 \quad x \leq 0 \text{ and } x \geq a$$

So, the potential of particle can be defined as

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & x \leq 0 \text{ and } x \geq a \end{cases}$$

Since TIME INDEPENDENT ONE-DIMENSIONAL SCHRÖDINGER'S EQUATION IS

$$\frac{\partial^2}{\partial x^2} \psi(x) + \frac{2m}{\hbar^2} [E - V(x)] \psi(x) = 0$$

Within the box $V(x) = 0$

$$\therefore \frac{\partial^2}{\partial x^2} \psi(x) + \frac{2m}{\hbar^2} E \psi(x) = 0 \quad \text{----- (1)}$$

$$\frac{\partial^2}{\partial x^2} \psi(x) + k^2 \psi(x) = 0 \quad \text{----- (2)}$$

$$\text{Here } k^2 = \frac{2m}{\hbar^2} E$$

$$\text{or } k = \sqrt{\frac{2m}{\hbar^2} E}$$

The general solution for equation (2) is

$$\psi(x) = A \sin kx + B \cos kx \quad \text{----- (3)}$$

Here A and B are constants and can be determine by using boundary condition so using boundary condition on equation (3)

$$\psi = 0 \quad \text{at} \quad x = 0 \quad \text{PUTTING IN EQUATION 3}$$

$$0 = 0 + B \quad B = 0$$

$$\psi(x) = A \sin kx \quad \text{----- (4)}$$

Again, from boundary condition

$$\psi(x) = 0 \quad \text{at} \quad x = a$$

$$0 = A \sin ka \quad \text{or} \quad \sin ka = 0 \quad \text{or} \quad ka = \sin^{-1} 0$$

$$ka = n\pi \quad \text{or} \quad k = \frac{n\pi}{a}$$

$$\psi(x) = A \sin \frac{n\pi}{a} x \quad \text{-----(5)}$$

Now since we know $k^2 = \frac{2m}{\hbar^2} E$ and $k = \frac{n\pi}{a}$

$$\frac{n^2 \pi^2}{a^2} = \frac{2mE}{\hbar^2}$$

$$E = \frac{n^2 \pi^2 \hbar^2}{2m a^2} \quad \text{This is known as ENERGY EIGEN FUNCTION.}$$

So

$$E_1 = \frac{\pi^2 \hbar^2}{2m a^2} \quad E_2 = \frac{4 \pi^2 \hbar^2}{2m a^2} \quad E_3 = \frac{9 \pi^2 \hbar^2}{2m a^2}$$

$$E_1 : E_2 : E_3 \dots \dots \dots = \frac{\pi^2 \hbar^2}{2m a^2} : \frac{4 \pi^2 \hbar^2}{2m a^2} : \frac{9 \pi^2 \hbar^2}{2m a^2} : \dots \dots \dots$$

$$E_1 : E_2 : E_3 \dots \dots \dots = 1 : 4 : 9 : 16 :$$

Now to calculate the value of constant A:

$$\int_{-\infty}^{\infty} |\psi_n(x)|^2 dx = 1 \quad \text{(NORMALIZATION CONDITION)}$$

$$\int_0^a \left| A \sin \frac{n\pi}{a} x \right|^2 dx = 1$$

$$A^2 \int_0^a \sin^2 \frac{n\pi}{a} x dx = 1 \quad \because \quad \sin^2 \theta = \frac{1}{2} (1 - \cos \theta)$$

$$\frac{A^2}{2} \int_0^a \left[1 - \cos \frac{2n\pi}{a} x \right] dx = 1$$

$$\frac{A^2}{2} \left[\int_0^a 1 \cdot dx - \int_0^a \cos \frac{2n\pi}{a} x dx \right] = 1$$

$$\frac{A^2}{2} \left[a - \frac{2n\pi}{a} \left[\sin 2n\pi \frac{a}{a} - \sin 0 \right] \right] = 1$$

$$\frac{A^2}{2} \left[a - \frac{2n\pi}{a} [0 - 0] \right] = 1$$

$$\frac{A^2}{2} a = 1$$

Or $A = \sqrt{\frac{2}{a}}$

Therefore $\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$ THIS EQUATION IS KNOWN AS EIGEN WAVE FUNCTION.

SYLLABUS

UNIT 3

COHERENCE AND OPTICAL FIBRE

- Spatial and temporal coherence, coherence length, coherence time and Q factor for light, Visibility as a measure of coherence, Spatial coherence and size of source, Temporal coherence and spectral purity.
- Optical fiber as optical wave guide, Numerical aperture, Maximum angle of acceptance and Application of Optical fiber.

COHERENCE:

When two or more light waves reaches to a point in space with a phase difference is about to zero and minimum path difference from source and having same wavelength, frequency and amplitude then these lights wave will be coherent to each other and this nature of light waves is known as COHERNCE.

The sources which emits coherent light wave to each other known as COHERNCE SOURCES.

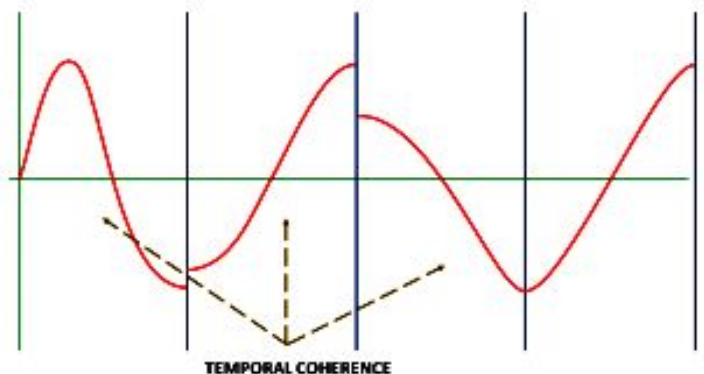
TYPES OF COHERENCE:

1. TEMPORAL COHERENCE
2. SPATIAL COHERENCE

TEMPORAL COHERENCE:

In TEMPORAL COHERENCE there is a definite relationship between the phase of waves at a given instant of time.

However, there is no real source of light which produces an ideal sinusoidal wave for longer time. An excited electron emits photon accurately sinusoidal wave for 10^{-10} sec. After this time interval wave changes its phase abruptly.



COHERNCE LENGTH AND COHERENCE TIME:

The average time interval for which definite phase relationship exist is known as COHERENCE TIME (τ) and the distance traveled by wave is called as COHERNCE LENGTH (L).

$$L = \tau \times c$$

Where c = The speed of light
 τ = Coherence time L = Coherence Length
Since we know PURITY OF SPECTRAL LINES Q is

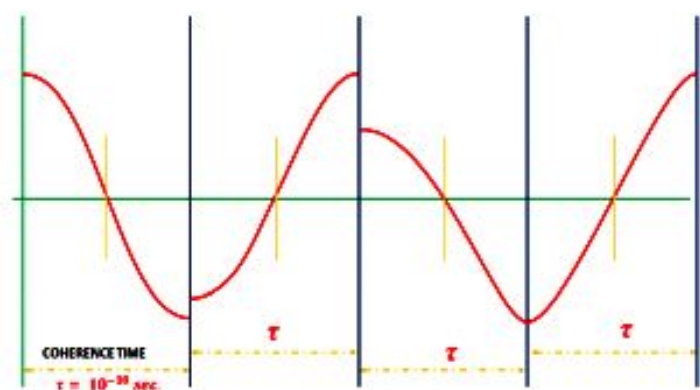
$$Q = \frac{\lambda}{\Delta\lambda}$$

And Coherence time is

$$\tau = \frac{1}{\Delta\nu}$$

$$\therefore L = \frac{c}{\Delta\nu}$$

$$\text{And } \therefore \nu = \frac{c}{\lambda}$$



Differentiating we will get

$$\Delta v = -\frac{c}{\lambda^2} \Delta \lambda$$

∴ Coherence time

$$\tau = \frac{\lambda^2}{c \Delta \lambda}$$

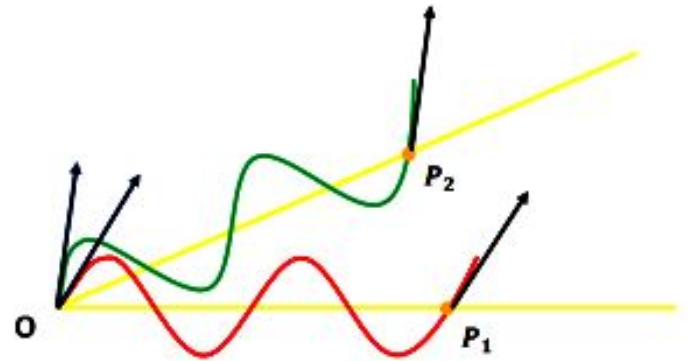
And purity factor

$$Q = \frac{\lambda}{\Delta \lambda}$$

SPATIAL COHERENCE

Spatial Coherence is a definite phase relationship between two or more waves at two or more points at certain distance from source.

Let us consider light wave emitted from a point source and two point P_1 and P_2 are two points are equidistance from the source S . If the waves reach at points in same phase then these two points will have perfect spatial coherence.



SPATIAL COHERENCE AND SIZE OF SOURCE

As shown in figure there are two slits S_1 and S_2 which are at equidistance from source S . The distance between S_1 and S_2 is y . The maximum value of y for source S_1 and S_2 remains coherent is

Since we know for diffraction $a \sin \alpha = n \lambda$

If angle is very small then $\alpha \approx n \lambda$

$$\alpha \approx n \frac{\lambda}{a} \quad n = 1$$

$$\alpha \approx \frac{\lambda}{a} \quad \text{-----(1)}$$

from geometry $\theta = \angle S_1 S S_2 = \frac{y}{x}$

$$\theta = \frac{y}{x} \quad \text{-----(2)}$$

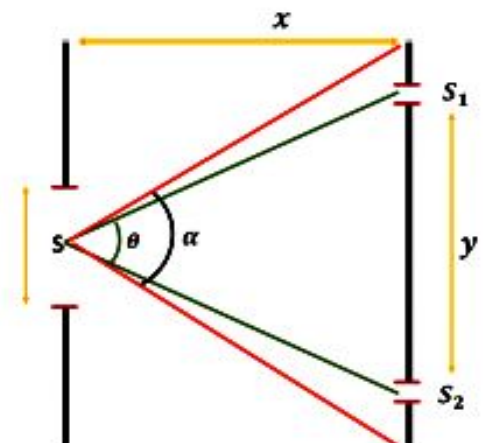
Now for slits S_1 and S_2 be COHERENT

$$\alpha \geq \theta \quad \theta \leq \alpha$$

From equation (1) and (2)

$$\frac{y}{x} \leq \frac{\lambda}{a} \quad y \leq \frac{\lambda}{a} \times x$$

$$y_{max} = \frac{\lambda}{a} \times x \quad \text{or} \quad L = \frac{\lambda}{a} \times x$$



for circular aperture

$$L_w = 1.22 \frac{\lambda}{\theta}$$

VISIBILITY AS A MEASURE OF COHERENCE:

The visibility of coherent sources can be defined as the ratio of difference between maximum intensity and minimum intensity to the sum of these intensities i.e.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

This ratio is a measure of the degree of coherence of light waves that produce the interference fringes. If two waves of equal intensities superimpose in their entire length, they would result in zero minimum intensity so we can say

$V = 1$ Such waves are said to be in perfect coherence with each other and result in maxima and minima of the highest degree of contrast.

If two waves do not overlap at all then $I_{max} = I_{min}$ then $V = 0$

Then no fringes are seen and the waves do not have any coherence.

Now let us consider two waves each of intensity I_0 superimpose to each other. Each of them consists of coherent part I_c and incoherent part I_{inc} then

$$I_c = C I_0 \quad \text{and} \quad I_{inc} = (1 - C) I_0$$

Here $C = \text{Degree of coherence}$

When waves superimpose the coherent part should interfere adding there by their amplitude.

Since we know

For coherent part resultant amplitude = $A + A = 2A$

$$\text{And } I = (2A)^2 \quad \text{or} \quad I_{max} = 4 A^2 \quad \text{or} \quad I_{max} = 4 I_c$$

$$I_{max} = 4 C I_0$$

Similarly for incoherent part

$$I_{min} = I_{inc} + I_{inc} \quad I_{min} = 2 I_{inc}$$

$$I_{min} = 2 (1 - C) I_0$$

$$\therefore V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad \therefore V = \frac{4 C I_0 - 2 (1 - C) I_0}{4 C I_0 + 2 (1 - C) I_0} \quad V = \frac{4 C I_0}{4 I_0}$$

$$V = C$$

OPTICAL FIBER

INTRODUCTION:

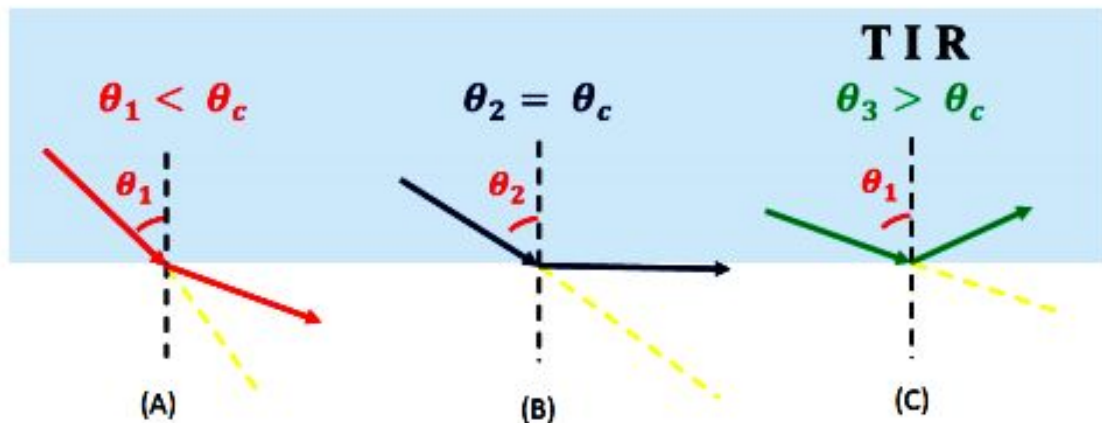
Fiber optics deals with light propagation through thin glass fibers. Fiber optics is an important and revolutionary communication system to transmit voice and digital data signals from one place to another. The transmission of light along the thin glass fiber by using phenomena TOTAL INTERNAL REFLECTION (TIR).

BASIC PRINCIPLE OF OPTICAL FIBER:

The basic principle of optical fiber communication is transmission of optical signal using total internal reflection.

TOTAL INTERNAL REFLECTION:

When the light ray travels from denser medium to rarer medium the refracted ray bends away from the normal. When the angle of incidence is greater than the critical angle, the refracted ray reflects into the same medium. This phenomenon of reflection of light ray from a refractive surface is called as TOTAL INTERNAL REFLECTION.



CONDITION OF TOTAL INTERNAL REFLECTION:

- (1) Light should travel from denser to rarer medium
- (2) Angle of incidence should be greater than critical angle ($\theta > \theta_c$)

OPTICAL FIBER

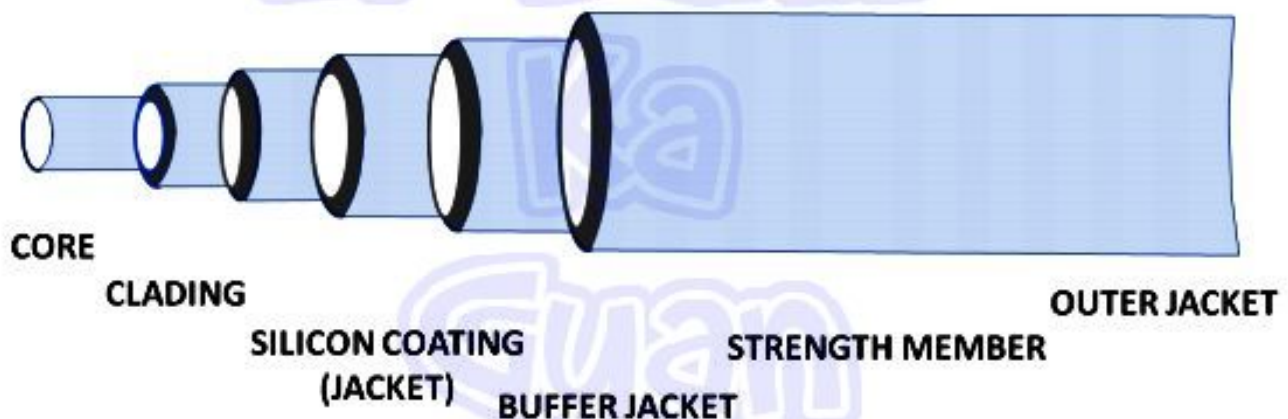
Optical fiber is a technology that uses glass, plastic, threads or fibres to transmit data. A fibre optic cable consists of a bundle of glass threads which are protected by the outer covering of several jackets.

Optical fibres use light to carry digital signals and is based on the concept of total internal reflection. The digital signal that is carried by the light is reflected inside the optical cable and hence transfers the information.

For a particular case of an optical fibre whose core is made of glass which is bounded by a plastic cladding the critical angle is 82° . Therefore the light when hits the plastic cladding at an angle more than 82° would be reflected back in the same medium, i.e., back to the glass core.

CONSTRUCTION: The optical fibre consists of three basic parts

1. CORE
2. CLADDING
3. JACKETS



CORE: Core is the innermost part of optical fibre basically made of glass or plastic and of diameter of $80\mu\text{m} - 100\mu\text{m}$. Signal transmission is performed in core.

CLADDING: It is coaxial cylindrical tube of diameter of $125\mu\text{m} - 150\mu\text{m}$. made of material having lower refractive index than core.

JACKET: The outer cylindrical coaxial tube made of plastic or polymer provided for protection from environmental changes such as temperature, humidity, electrical and magnetic field or jerks etc.

WORKING PRINCIPLE: When a light ray is incident at one end of the core of the fibre, because of the small radius of fibre light ray going into the core makes an angle greater than the critical angle and total internal reflection takes place and light will be reflected back into the core. And so light ray repeated

multiple reflections and propagates along the fibre from one end to the other end, even if light is put in curved zig-zag shape of fibre. Due to very high reflectivity at the core to cladding boundary, light can be transmitted without any appreciable loss. In silica glass fibres more than 95% of the light is transmitted over a distance of 1 Km. and due to this light to be transmitted over distances of nearly 250 Km without significant loss.

To transmission of signal through optical fibre

- (i) The core material should have higher refractive index (μ_{core}) than of the cladding material (μ_{clad}). i.e.

$$\mu_{core} > \mu_{clad}$$

- (ii) The angle of incidence should be greater than the critical angle. i.e.

$$\theta_i > \theta_c$$

CLASSIFICATION OF OPTIAL FIBRE:

There are several types of optical fibre available, on the basis of their uses we can classify optical fibre in following types

BASED ON REFRACTIVE INDEX OF CORE MEDIUM

1. STEP INDEX OPTICAL FIBRE
2. GRADDED INDEX OPTICAL FIBRE

BASED ON MODE OF TRANSMISSION

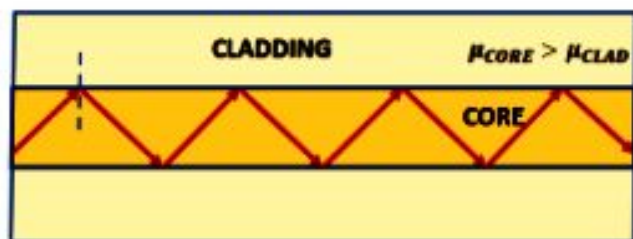
1. MONO (SINGLE) MODE OPTICAL FIBRE
2. MULTIMODE OPTICAL FIBRE

STEP INDEX OPTICAL FIBRE: In Step Index fibers the refractive index of the core medium is uniform and undergoes an abrupt change at the interface of core and cladding.

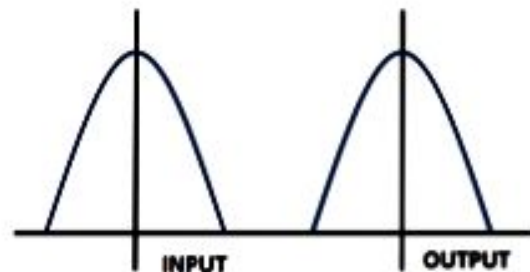
The transmitted optical signal will cross the fiber axis during every reflection at the core cladding interface (Boundary). A light wave, which travel along the core and meets the cladding at an angle greater than the critical angle (θ_c) will be gone through the phenomena total internal reflection. This reflected ray will then meet the opposite surface of the cladding, again at the same angle i.e. greater than critical angle and so is again total internally reflected. So, the light wave is transmitted along the fibre core in a zig-zag path.

STEP INDEX MONOMODE OPTICAL FIBRE: When a single light wave propagate through a

step index optical fibre in a single time then this kind of fibre is known as Step Index Monomode Optical Fibre.

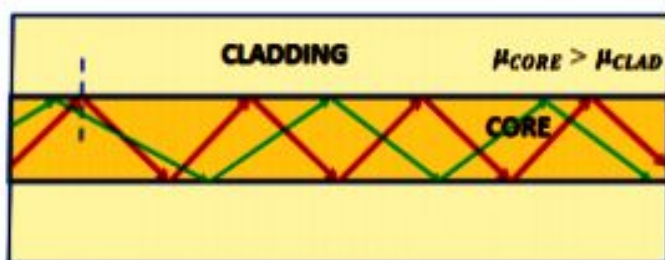


STEP INDEX MONO MODE OPTICAL FIBER

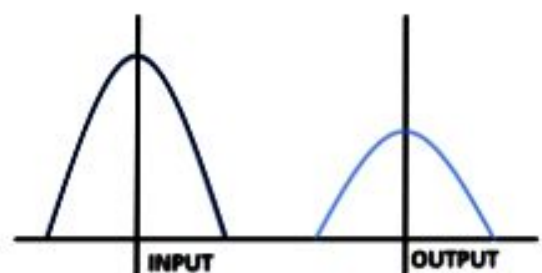


STEP INDEX MULTIMODE OPTICAL FIBRE: When multiple light waves of different frequency and energy propagate along the step index optical fibre then this kind of optical fibre is known as Multimode Optical Fibre.

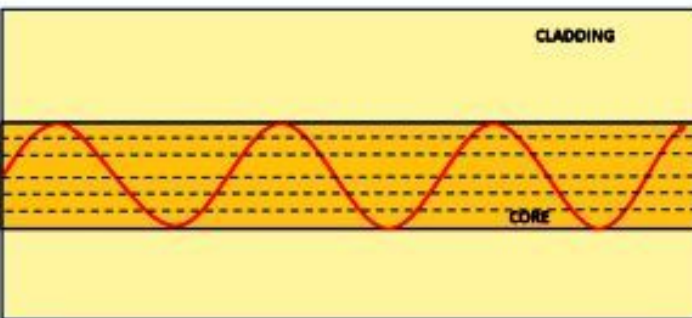
The various light waves, travelling along the core, will have propagation paths of different length, so they will take different time to reach at another end (receiver). Then a distortion is produced which is called transit- time dispersion. Due to this distortion, the variations of successive pulse of light may overlap into each other causing distortion of the information being carried.



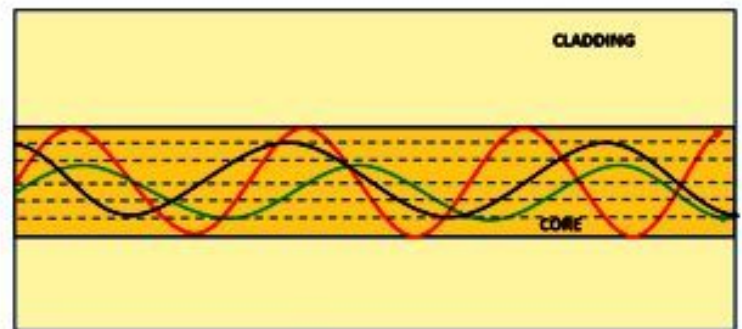
STEP INDEX MULTI MODE OPTICAL FIBER



GRADED INDEX OPTICAL FIBRE: The graded index optical fibre has refractive index of core decreases in a parabolic manner from a maximum value at the centre of the core to a constant value at the core-cladding interface.



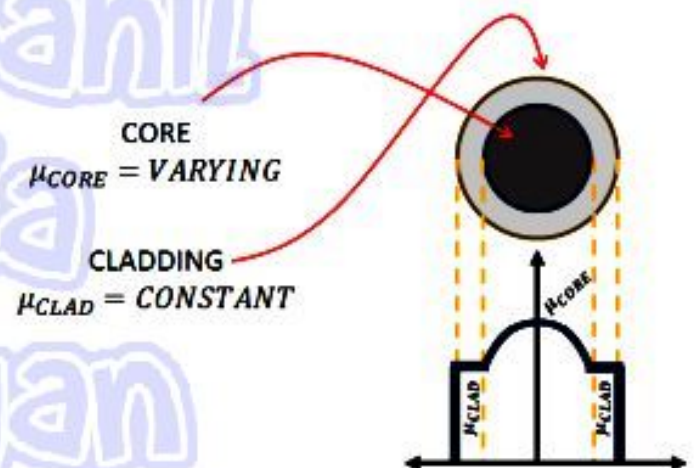
GRADED INDEX MONO MODE OPTICAL FIBER



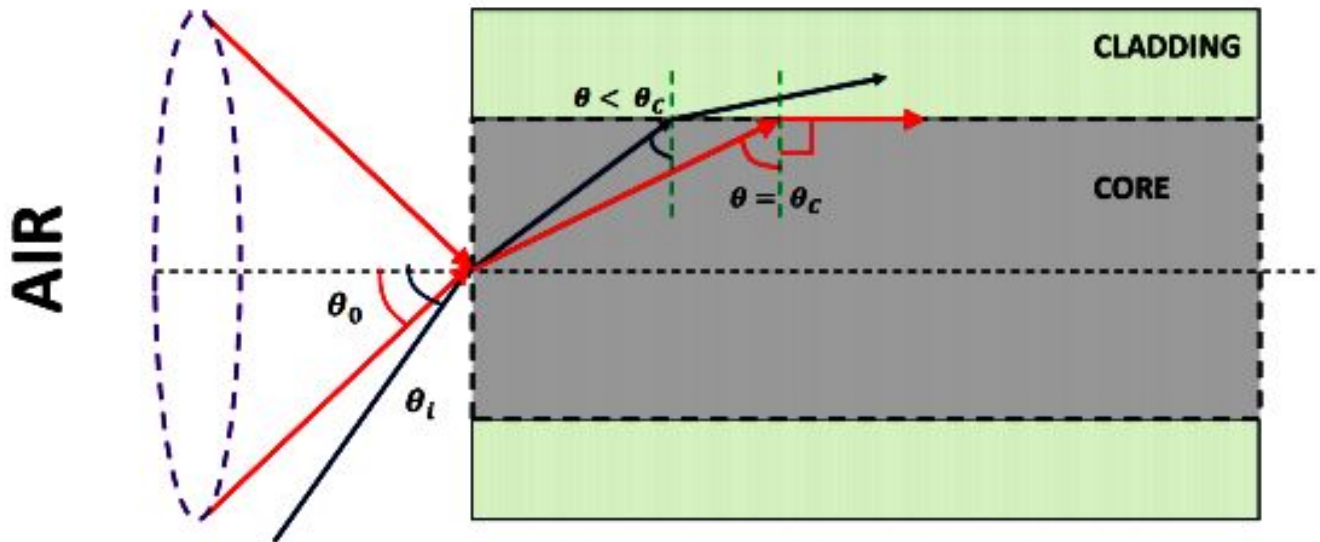
GRADED INDEX MULTI MODE OPTICAL FIBER

To understand the propagation of signals through graded index optic fibre (GRIN), consider individual light wave is being gradually refracted in the graded index core instead of being TIR by the cladding. So the waves travelling at different incidence angle will travel a different distances from the horizontal central axis before being TIR from any layer of graded core medium.

It is clear from diagrams that signals with larger incidence angles travel more path than those signal with smaller angles. But as we know that the decrease of refractive index allows a higher velocity of propagation. So, all the waves will reach a given point along the fibre at virtually same time.



ACCEPTANCE ANGLE AND NUMERICAL APERTURE:



From figure

Applying laws of refraction at point A

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\frac{\sin \theta_0}{\sin(90 - \theta_c)} = \frac{\mu_{core}}{\mu_0}$$

$$\mu_0 \sin \theta_0 = \mu_{core} \sin(90 - \theta_c)$$

$$\mu_0 \sin \theta_0 = \mu_{core} \cos \theta_c \quad \text{-----(1)}$$

$$\mu_0 \sin \theta_0 = \mu_{core} \sqrt{(1 - \sin^2 \theta_c)} \quad \text{-----(2)}$$

Applying laws of refraction at point B

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

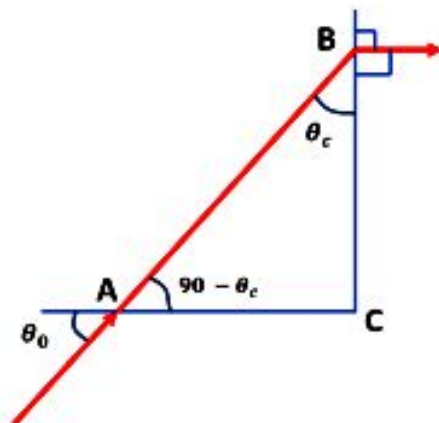
$$\text{Here: } i = \theta_c \quad r = 90^\circ$$

$$\mu_2 = \mu_{clad} \quad \mu_1 = \mu_{core}$$

$$\frac{\sin \theta_c}{\sin 90^\circ} = \frac{\mu_{clad}}{\mu_{core}}$$

$$\text{or} \quad \mu_{core} \sin \theta_c = \mu_{clad} \quad \text{-----(3)}$$

$$\text{Or} \quad \sin \theta_c = \frac{\mu_{clad}}{\mu_{core}} \quad \text{-----(4)}$$



Putting this value in equation (2)

$$\mu_0 \sin \theta_0 = \mu_{core} \sqrt{1 - \left(\frac{\mu_{clad}}{\mu_{core}}\right)^2}$$

$$\sin \theta_0 = \frac{\mu_{core}}{\mu_0} \sqrt{\frac{\mu_{core}^2 - \mu_{clad}^2}{\mu_{core}^2}}$$

$$\sin \theta_0 = \frac{\mu_{core}}{\mu_0} \frac{\sqrt{\mu_{core}^2 - \mu_{clad}^2}}{\mu_{core}}$$

$$\sin \theta_0 = \frac{\sqrt{\mu_{core}^2 - \mu_{clad}^2}}{\mu_0}$$

If the optical fibre is placed in air $\mu_0 = 1$

$$\sin \theta_0 = \sqrt{\mu_{core}^2 - \mu_{clad}^2}$$

Or

$$\theta_0 = \sin^{-1} \sqrt{\mu_{core}^2 - \mu_{clad}^2}$$

NUMERICAL APERTURE

$$N.A. = \sin \theta_0 = \sqrt{\mu_{core}^2 - \mu_{clad}^2}$$

Numerical aperture is a measure of gathering of light in optical fibre.

$$N.A. = \sqrt{\mu_{core}^2 - \mu_{clad}^2}$$

$$N.A. = \sqrt{(\mu_{core} + \mu_{clad})(\mu_{core} - \mu_{clad})}$$

$$N.A. = \sqrt{2 \mu_{core} (\mu_{core} - \mu_{clad})}$$

$$N.A. = \sqrt{2 \mu_{core}^2 \frac{(\mu_{core} - \mu_{clad})}{\mu_{core}}}$$

$$N.A. = \mu_{core} \sqrt{2 \Delta}$$

here $\mu_{core} + \mu_{clad} \approx 2 \mu_{core}$

And $\frac{(\mu_{core} - \mu_{clad})}{\mu_{core}} = \Delta = \text{fractional difference of index}$

SYLLABUS

UNIT 4

LASER

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- ✓ **INTRODUCTION TO LASER**
- ✓ **PROCESSES REQUIRED IN LASER ACTION**
 - ABSORPTION**
 - SPONTANEOUS EMISSION**
 - STIMULATED EMISSION**
- ✓ **RELATION BETWEEN EINSTEIN'S COEFFICIENTS**
- ✓ **PRINCIPLE OF LASER**
- ✓ **He-Ne LASER**
 - CONSTRUCTION**
 - WORKING**
- ✓ **SEMICONDUCTOR LASER**
 - CONSTRUCTION**
 - WORKING**
- ✓ **Q- SWITCHING AND MODE LOCKING**
- ✓ **CHARACTERISTICS AND APPLICATION OF LASER**

LASER name stand for

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The basic principle involved in LASER ACTION is the phenomenon of stimulated emission which was predicted by Einstein in 1971. The phenomena will be explained later in this chapter.

CHARACTERISTICS OF LASER

The following characteristic which distinguishes a laser beam from an ordinary light are:

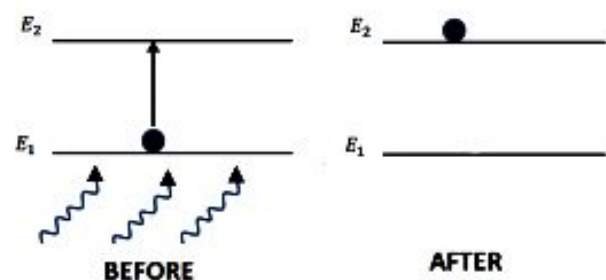
1. Highly coherence
2. Highly intense
3. Highly directional
4. Highly monochromatic

THEORY OF LASER:

Action of LASER is based on quantum theory of light. Production of LASER light is a particular consequence of interaction of radiation with matter. As we know the transition of energy in a system (atom) can occur in two stages. In one, the atom can absorb energy and go to higher energy level. And in another one the atom or particle come back to the lower energy level by radiating energy. So an external radiation is incident on the medium under normal state conditions, there are three processes take places which are:

1. Absorption
2. Spontaneous emission
3. Stimulated (induced) emission

1. ABSORPTION PROCESS: let us consider two energy levels in an atom. The lower energy level is denoted by E_1 and higher energy level is denoted by E_2 . As the atom or particle initially in E_1 and can be raised to E_2 by absorbing some energy incident on it in the form of light (photons) as shown in fig.



The probability of this transition depends on the two energy levels under consideration. It also depends on the number of photons incident on system.

$$\text{So } P_{12} \propto u(\nu)$$

$$P_{12} = B_{12} u(\nu)$$

So the rate of absorption will be

$$R_{12} = N_1 B_{12} u(\nu) \quad \text{-----(1)}$$

Here N_1 = Number of atoms (Particle) in E_1

R_{12} = Rate of Number of atom going from 1st energy level to 2nd energy level.

B_{12} = Einstein co-efficient of absorption.

2. SPONTANEOUS

EMISSION: Now when atom (particle) is excited to higher energy level, has a small life time (10^{-8} sec.) so it can jump from higher energy level to its ground state (i.e. E_1) by emitting a photon of frequency (ν). This is called as spontaneous emission. The rate of this process depends on the number of particle in excited state only as the probability is 100% (=1). Therefore

$$P_{21} = A_{21}$$

$$R_{21} = N_2 A_{21} \quad \text{-----(2)}$$

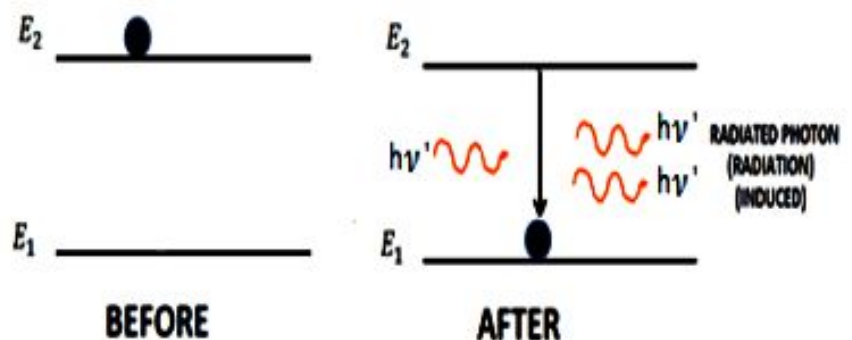
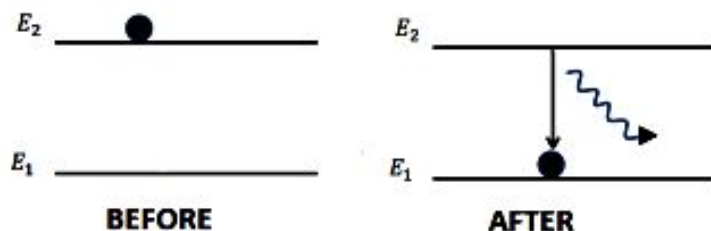
Here N_2 = Number of atoms (Particle) in E_2

R_{21} = Rate of Number of atom going from 2nd energy level to 1st energy level.

A_{21} = Einstein co-efficient of spontaneous emission.

3. STIMULATED EMISSION:

As per Einstein when energy incident on system it is not essential that it may always be absorbed by ground state atom (Particles), It can also interact with an atom (particle) in excited state and induce that atom (particle) to emit a new photon. This process of emission is known as



induced or stimulated emission.

To understand this process, when a particle is in excited state after mean life time a new photon of incident energy interact with this excited particle electron excited to another energy level or hold at a metastable energy level (i.e. equal to incident energy) for a very short duration of 10^{-8} sec. Jump back to its ground state by radiating coherent radiation with other particle in that energy level (or metastable state). The rate of induced emission will be proportional to the incident energy and number of particle present in excited energy level.

$$(P_{21})_I \propto u(\nu)$$

$$(P_{21})_I = B_{21}u(\nu)$$

$$(R_{21})_I = N_2 B_{21} u(\nu) \quad \text{-----(3)}$$

Here N_2 = Number of atoms (Particle) in E_2

$(R_{21})_I$ = Rate of Number of atom going from 2nd energy level to 1st energy level.

B_{21} = Einstein co-efficient of spontaneous emission.

RELATION BETWEEN EINSTEIN'S COEFFICIENT

(THRESHOLD CONDITIONS FOR LASER ACTION):

As we know

$$R_{12} = N_1 B_{12} u(\nu) \quad \text{-----(1)}$$

$$R_{21} = N_2 A_{21} \quad \text{-----(2)}$$

$$(R_{21})_I = N_2 B_{21} u(\nu) \quad \text{-----(3)}$$

And in thermal equilibrium

Rate of absorption = total rate of emission

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$R_{12} = R_{21} + (R_{21})_I$$

$$N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

Dividing numerator and denominator by $N_2 B_{21}$

$$u(\nu) = \frac{\frac{N_2 A_{21}}{N_2 B_{21}}}{\frac{N_1 B_{12}}{N_2 B_{21}} \frac{N_2 B_{21}}{N_2 B_{21}}}$$

$$u(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \quad \text{-----(4)}$$

Now since we know Maxwell's Boltzmann distribution law

$$N \propto e^{-\frac{E}{kT}}$$

$$\frac{N_1}{N_2} = e^{-\left(\frac{E_1 - E_2}{kT}\right)}$$

$$\frac{N_1}{N_2} = e^{\left(\frac{h\nu}{kT}\right)} \quad \text{-----(5)}$$

Putting this value in equation (4)

$$u(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}}{B_{21}} e^{\left(\frac{h\nu}{kT}\right)} - 1} \quad \text{-----(6)}$$

The above equation is an expression of spectral energy density can be compared with Planck's radiation law, according to which

$$u(\nu) = \frac{8\pi h\nu^3/c^3}{e^{\left(\frac{h\nu}{kT}\right)} - 1}$$

So we can say

$$\frac{A_{21}}{B_{21}} = 8\pi \frac{h\nu^3}{c^3} \quad \text{-----(7)}$$

$$\frac{B_{12}}{B_{21}} = 1 \quad \text{or}$$

$$B_{12} = B_{21} \quad \text{-----(8)}$$

So it results in that in the presence of an incident photon absorption and induced emission are equal probable.

Now comparing the probability of spontaneous emission and stimulated emission.

$$\frac{(P_{21})_s}{(P_{21})_i} = \frac{A_{21}}{B_{21} u(\nu)} = (e^{h\nu/kT} - 1) \quad \text{-----(9)}$$

Now applying two conditions:

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Gyan

1. if $h\nu \gg kT$, $(P_{21})_s \gg (P_{21})_i$

In this condition spontaneous emission will be much more probable than induced emission. This is not useful for LASER production.

2. if $h\nu \ll kT$ then $(e^{h\nu/kT} - 1) \approx h\nu/kT$ in this case $(P_{21})_s < (P_{21})_i$ so the induced (stimulated) emission is in action so such situation make it to achieve LASER action.

BASIC PRINCIPLE OF LASER

As we know Maxwell-Boltzmann law, the ratio of rate of emission and rate of absorption is given by

$$\frac{\text{Rate of Emission}}{\text{Rate of Absorption}} = \frac{R_{21}}{R_{12}} = \frac{N_2(A_{21} + B_{21}u(\nu))}{N_1 B_{12} u(\nu)}$$

Since $B_{12} = B_{21}$

$$\frac{R_{21}}{R_{12}} = \left(\frac{A_{21}}{B_{12} u(\nu)} + 1 \right) \times \frac{N_2}{N_1}$$

So we can say for LASER action $\left(\frac{A_{21}}{B_{12} u(\nu)} + 1 \right) = 1$ for this $\left(\frac{A_{21}}{B_{12} u(\nu)} \right) \approx 0$ so $A_{21} \ll B_{12} u(\nu)$

- I. It means Probability of spontaneous emission should be very less than Probability of stimulated emission.

$$\frac{R_{21}}{R_{12}} = \frac{N_2}{N_1}$$

- II. The emission rate should be larger than the absorption rate.

These conditions can be achieved by using following steps:

1. The number of atoms in higher energy state should be higher than number of atoms in lower energy states. This condition is known as population inversion. Due to population inversion stimulated emission can be achieved (1st condition of LASER ACTION).
2. The system of method by which we can achieve to population inversion is known as PUMPING METHOD. For population inversion pumping is continuously required. There are several methods of pumping such as

- a. Optical Pumping (Ruby laser)
- b. Electrical discharge (He-Ne laser)
- c. Flash discharge (Ruby laser)
- d. chemical pumping
- e. Gas dynamic pumping
- f. Atom-atom collision etc.

3. Due to the short life time of an atom in higher state, atoms cannot remain in the higher excited state so long and rapidly decay to lower states. As the electron drop back, some of them are being trapped in the intermediate state that is called as METASTABLE STATE due to a non-radiative transition and excess energy is absorbed by the lattice. The life time of an atom in metastable state is of the order of 10^{-3} sec. which is very larger than the life time of the atom in the excited state. If the pumping

method is intense enough we may obtain a population inversion, i.e. more atoms in metastable state than in the ground state (2nd condition of LASER ACTION).

MAIN COMPONENT OF LASER

- (A) ACTIVE MEDIUM (B) PUMPIG SOURCE (METHOD)
(C) OPTICAL RESONATOR

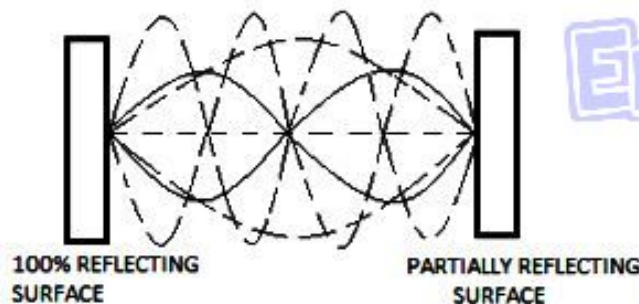
(A) ACTIVE MEDIUM: The active medium can be solid, liquid or gas in which LASER action is performed. It should be capable of amplifying radiation (light waves). Only a small fraction of particular medium is responsible to stimulate emission. These fractions are known as active centre and remaining part of medium merely support active centre.

(B) PUMPIG SOURCE (METHOD) : Pumping source or method is the mechanism of exciting atoms from the lower energy state to a higher energy state by supplying energy from an external energy source. Most commonly it is known as amplification process.

There are several pumping methods such as

- | | | |
|---------------------------------|------------------------------------|------------------------|
| (I) Optical (Discharge) Pumping | (ii) Electrical Discharge | (iii) Chemical Pumping |
| (iv) Mechanical Pumping | (V) Inelastic Atom- Atom Collision | |
| (Vi) Gas Dynamic Pumping | (Vii) Gas Dynamic Pumping | |

(C) OPTICAL RESONATOR: In a LASER, the active medium is enclosed in an optical cavity that is made up of two parallel surfaces, one of which is perfectly reflecting and the other surface is partially reflecting. In this resonant cavity, the intensity of photons is raised through stimulated emission process.



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Ka
Gyan

IMPORTANT FEATURES IN LASER ACTION:

METASTABLE STATE: There are certain energy states whose life times are much longer than the life times of usual short lived excited states. Such relatively long lived states are called metastable states whose average life time period is about 10^{-3} sec. Due to longer life time τ_c of the metastable states the Einstein coefficient of spontaneous emission $A_{21} \propto \frac{1}{\tau_c}$ is comparatively small. Thus

spontaneous emission from metastable state is forbidden to some degree. So there are probabilities of stimulated emission increased.

POPULATION INVERSION: Since in thermal equilibrium the number of atoms in higher energy states is considerably smaller than the number of atoms in lower energy states, hence there is very less stimulated emission compared with the spontaneous emission. But by using metastable state the number of atoms in higher energy state can be enhanced. This situation plays very important role in generation of LASER. So the situation in which the number of atoms in then higher energy state exceeds than that in the lower energy state is known as population inversion.

PRODUCTION OF LASER

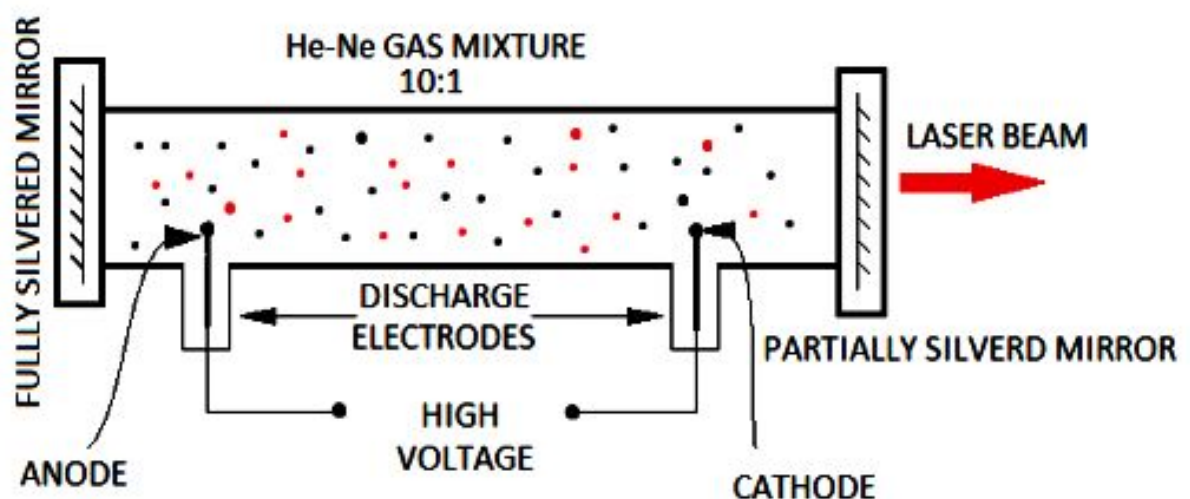
Now a days different kinds of LASERS are available, the most common being in a digital communications. Virtually every house now has at least on in their CD/DVD players and recorders. Some LASERS can change color since they are tunable LASERS. The lasers now operate from the infrared to the ultraviolet regions. Moreover X-ray lasers are being developed using electron accelerators. The lasers now are available in the wide range such as solid state laser, gas lasers, semiconductor lasers, etc.

The most common LASERS are

- (a) He- Ne laser (b) Semiconductor laser

HELIUM- NEON LASER:

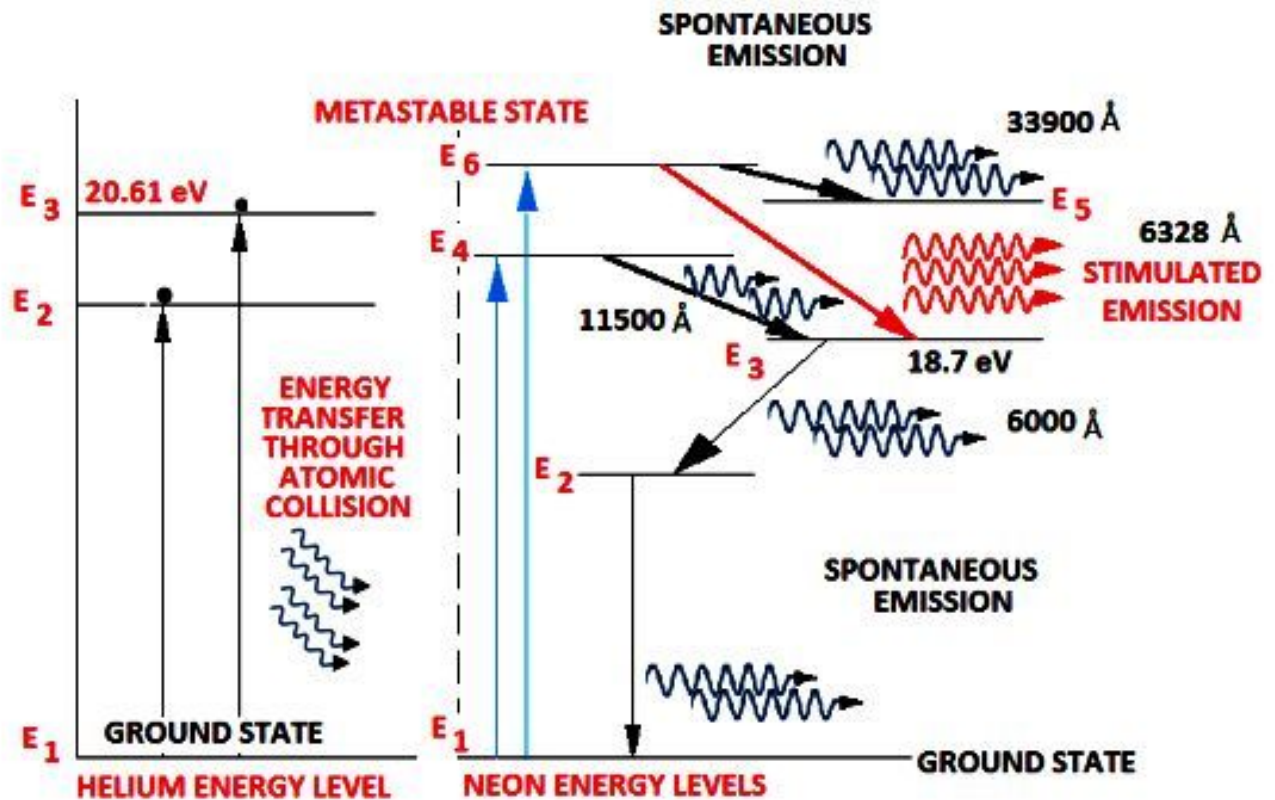
CONSTRUCTION: The He-Ne laser consists of a long and narrow quartz tube of 80-100 cm. and diameter of about 1 cm. In this laser system, quartz tube is filled with a mixture of helium and neon gases in the ration 10:1 respectively at a pressure of about 0.1mm of mercury. This mixture of gases acts as active medium. This system is enclosed between a pair of plane mirrors or a pair of concaves mirrors



so that an optical resonator system is formed. One of these mirrors is highly reflective and another one is partially reflective. The spacing between the two mirrors is equal to an integral multiple of half wavelengths of LASER light.

An electrical discharge (spark plug) is used as a pumping source. The electric discharge is produced in the gas mixture by means of electrode outside the tube connected to a source of high frequency AC.

WORKING: When an electric discharge takes place through the gas mixture, charges of this discharge collide with the He atoms in ground state and excite them to higher energy levels (E_2 and E_3) of He. E_2 and E_3 of He atom are metastable states and due to long life time before getting de excited the number of excited state atoms of He increases. Now these excited electrons of He collides with electron in ground state of Ne. These He electrons de-excite to ground state by transferring their energy with additional kinetic energy to ground state electrons of Ne. This results the excitation of Ne atoms to the level E_4 and E_6 . This process is highly probable due to longer life time of E_2 and E_3 of He atom.



This results population increase in E_4 and E_6 of Neon. Thus the purpose of He atoms is to help to achieve a condition of population inversion in the Ne atom.

When the state of population inversion is achieved then spontaneously emitted photon trigger laser action in any of the three transaction i.e. the transaction from E_6 to E_5 , E_4 to E_3 and E_6 to E_3 by emission of radiation of wavelengths 33900 Å, 11500 Å and 6328 Å respectively. The first two wavelengths are not in the visible region while the last transaction of 6328 Å gives the red light of the He-Ne laser.

These photons of wavelength 6328 Å travelling parallel to the axis bounces back and forth between the mirrors (resonator) and will stimulate another excited neon atom to emit another photon of same frequency and phase and the radiation gets amplified and emerge a red laser light. Other excited atoms get back to the ground state through collision with the walls of narrow quartz tube.

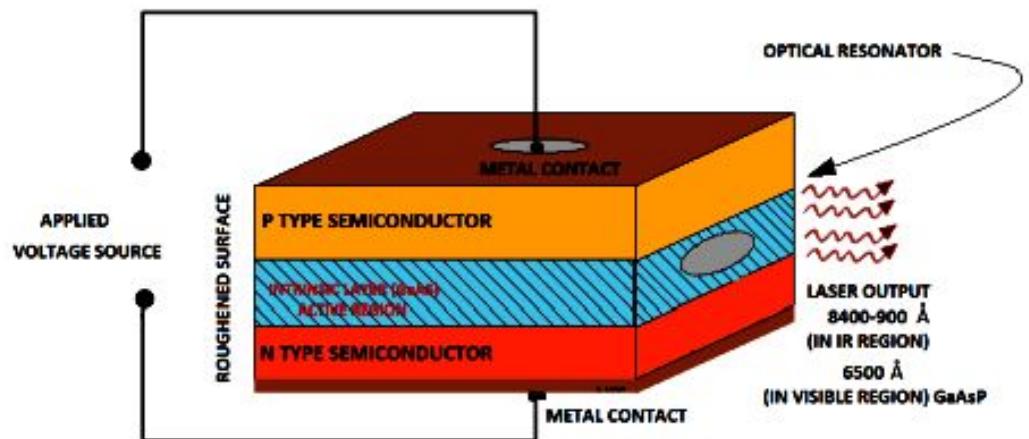
Typically power output of He-Ne laser lie between 1-50 mW of continuous wave for input about 5-10W.

SEMICONDUCTOR LASER:

Semiconductor laser is conventional laser. Laser action results through transitions associated with the properties of energy bands of the materials.

CONSTRUCTION: There are three basic requirements to achieve laser action:

1. Excitation of electrons or pumping from valence band to conduction band
2. Large concentration of electrons in conduction band and large concentration of holes in valence band so that the inverted charge carriers give sufficient stimulated emission to overcome the losses.



3. A resonant cavity to provide positive amplification.

The basic structure of a p-n junction laser is shown in figure. Both P and N sides of the junction are heavily doped so that the fermi levels are within the conduction and valence band edges respectively. (Typical GaAs laser are fabricated by diffusion of acceptor atoms as Zn forming the P-region into an N type as GaAs doped with Te with a carrier concentration of about 10^{18} cm^{-3}). A pair of parallel planes perpendicular to the plane of the junction are cleaved or polished to prepare RESONATOR. The two remaining sides are roughened to eliminate lasing in directions other than the main one. The structure is called Fabry –Perot cavity. The top and bottom faces are made a metal contact.

WORKING: The energy band diagram of p-n junction laser is shown in fig. The equilibrium state with zero bias and forward bias state with population inversion state is shown.

At equilibrium in the zero bias state, due to high doping the Fermi level lies in the valence band on the P-side and in the conduction band on the N-side. The state filled with electrons are shown by shaded regions. When a small forward bias is applied electrons start flowing from conduction band on N-side to the vacant sides on the P-side and holes start flowing to the N-side. Thus charge carriers are pumped by the dc voltage source into the junction region and a small forward current is produced. At this stage electron hole recombination causes spontaneous emission of photons and the device acts as a LED. As the applied voltage is increased the forward current through the junction increases. When this voltage is high enough the current reaches a threshold value and population is inverted on junction active region.

In this thin region a large concentration of electron in the conduction band and a large concentration holes in valence band gets developed. This narrow zone is called inverse region or active region. At threshold LASER action begins. The dependence of radiation output increase of current. The coherent radiation gets amplified in the cavity and is finally emitted from the optically plane reflecting ends.

Q SWITCHING: As we know laser is an oscillator, its resonator cavity is characterized by the quality factor Q.

Quality factor (Q) is defined as the ratio of the energy W stored in the system to the energy losses ΔW per cycle.

$$Q = 2\pi \times \frac{\text{Energy stored in the resonator}}{\text{Energy lost in a cycle}}$$

$$Q = 2\pi \times \frac{W}{\Delta W}$$

In case of laser, quality factor Q is very high and is of the order of 10^8 . Q- factor is a measure of the mirror losses in laser.

The power of laser can be increase by increase number of atoms in the active medium participating in stimulated emission. It is possible when high population inversion is established. When laser starts oscillations the population inversion drops to threshold value. So laser can be prevented from oscillating if the parallelism of the resonator mirrors is disturbed. If one of the end mirror is misaligned, it cannot reflect incident photons into the active medium and stimulated emission cannot take place.

Thus laser oscillations set in suddenly and the Q of the cavity is increased abruptly. All the energy stored in the cavity is emitted in a single pulse with peak power much higher than the laser could produce otherwise. The pulse lasts for a short time and depopulates the upper energy level quickly and the lasing action stops. This method of controlling the lase output power is called Q-switching method.

MODE LOCKING: Mode locking is another technique for producing high power, short duration pulses. Single mode operation can be achieved by forcing random modes to oscillate in phase so that it produces pulse of short duration and higher gain. This phenomenon is called mode locking.

There are several ways a laser can be mode locked

1. Active mode locking
2. Passive mode locking
3. Self-mode locking

1. Active mode locking: A physical device is placed in the cavity which modulates the frequencies of the cavity mode.

2. Passive mode locking: An intensity dependent loss mechanism placed in the cavity causes less intense radiation to be damped out, leaving only a single, intense pulse oscillating back and forth in the cavity.

3. Self-mode locking : The lasing medium itself has an intensity dependent index of refraction.

SYLLABUS

UNIT 5

ELEMENT OF MATERIAL SCIENCE

- ✓ **INTRODUCTION**
- ✓ **BONDING IN SOLIDS**
 - **IONIC BOND**
 - **COVALENT BONDS**
 - **METALLIC BONDS**
 - **HYDROGEN BOND**
 - **VANDER WAAL BONDS OR MOLECULAR BOND**
- ✓ **ENERGY BAND IN SOLIDS**
- ✓ **CLASSIFICATION OF SOLIDS ON THE BASIS OF ENERGY BAND GAP**
 - **INSULATORS**
 - **CONDUCTOR**
 - **SEMICONDUCTOR**
- ✓ **INTRINSIC SEMICONDUCTORS**
- ✓ **CONDUCTIVITY IN INTRINSIC SEMICONDUCTOR**
- ✓ **EXTRINSIC SEMICONDUCTORS**
 - **N- TYPE SEMICONDUCTORS**
 - **P- TYPE SEMICONDUCTORS**
- ✓ **HALL EFFECT**
- ✓ **HALL COEFFICIENT**

INTRODUCTION

A solid contains number of atoms packed closely together. When N atoms of the solid are well separated, then these atoms lead to N -fold degenerate levels of the solid. As the atoms approach one another to form a solid. This causes each of the levels to split into N distinct levels. It is the separation distance which specifies the amount of overlap that causes the splitting. The amount of splitting is different for different energy levels. It means the lowest levels remain almost unsplit. The reason is that the electrons in lower levels are the ones which are in inner subshells of the atoms. So they are not significantly influenced by the presence of nearby atoms. Since the potential barriers between the atoms are for them relatively high and wide, these electrons are localized in particular atoms, even when separation distance is very small. However, the electrons in the higher levels are the valence electrons and are not localized at all for small separation distance but they become part of the whole system.

BONDING IN SOLIDS:

There are various types of bonds existing between the atoms constituting a solid. These are produced by the forces between the atoms, ions and molecules. In a solid these atoms are in close proximity to one another. The interatomic separation is of the order of few angstroms ($\text{\AA}$) and the radius of an atom is also of the same order of magnitude.

There are five types of bonds :

- | | |
|--|--|
| 1. Ionic bond : (NaCl, CsCl, MgO) | 2. Covalent Bond : (C, Si, Ge) |
| 3. Metallic Bond: (Na, Al, Cu, Ag, Zn) | 4. Hydrogen Bond: (H_2O , HF) |
| 5. Vander Waals Bond: | |

IONIC BOND:

An ionic bond is formed between two different atoms, one electropositive and the other electronegative.

OR

In an ionic bond between two atoms one or more electrons are transferred from one atom to the other with the result that one of the atoms becomes positively charged ion and the other negatively charged ion.

The electrostatic attraction between these ions binds them to form a molecules.

E.g. Let us consider a case of NaCl. When Na and Cl atoms are brought together, the outer valence electron of the Na atom is transferred to the Cl atom which has seven valence electrons. This makes both these atoms to achieve more stable electronic configuration lowering the overall energy.

PROPERTIES

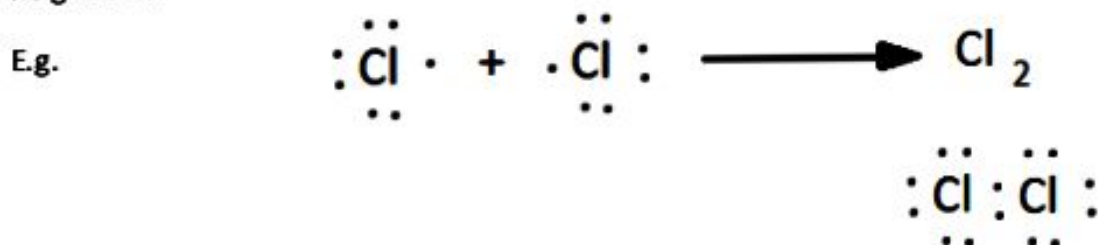
Ionic crystals have large cohesive energies and hence these crystals have cohesive energies and hence these crystals have higher melting points.

E.g. NaCl is 180 K cal./mol. and its melting point is about 800°C. In general their melting points vary from 700K to 300K.

The electrical and thermal conductivities of ionic crystals are much lower than the conductivities of metals. Most of these are diamagnetic however in some exceptional cases paramagnetic nature is found. These crystals are hard and brittle. These are usually transparent for frequencies up to the fundamental absorption frequency which lies in the UV region.

COVALENT BONDS:

The covalent bond which is sometimes referred to as valence bond or homopolar bond, is an electron-pair bond in which two atoms share two electrons. In the covalent bonding each atom of the element effectively completes its outer electron shell by sharing electrons with its neighbors.



Two unpaired electrons between the two chlorine atoms forming the covalent bond, thus belong to both and provide each atom with the stable configuration.

The number of covalent bonds that an atom of an element can form depends on how many unpaired electrons it has. This number can be given by the so called (8-N) rule where N is the number of the column in the periodic table containing the given element.

PROPERTIES

The covalent bond is a strong bond and so materials having this bond have high cohesive energy. Because of their rigid electronic structure the covalent crystals are extremely hard and difficult to deform. Like ionic crystals the covalent crystals are transparent to radiation of longer wavelengths. At higher frequencies the photons can excite the electrons in the valence band to the empty levels in the conduction band. The electrical conductivity of covalent crystals varies over a wide range. Some of these like diamond are excellent insulators, Ge and Si are semiconductors while tin has much higher conductivity approaching that of metals. These crystals are poor conductors of heat and are diamagnetic in nature.

METALLIC BONDS:

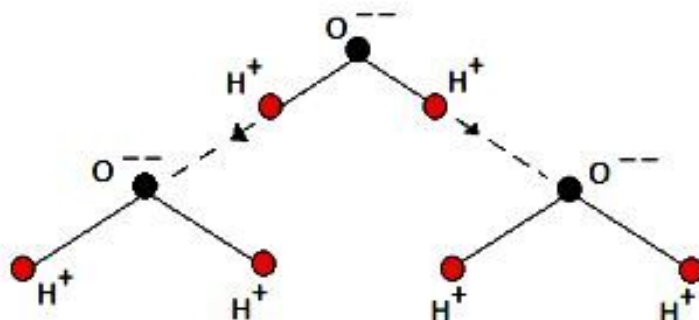
Metallic crystals are commonly known as metals. In the atoms of metals, the electrons in the outermost orbits are loosely bound as the ionization energy is low in the case of metals. These electrons are free to move around among all the atoms and are called free electrons or conduction electrons. This way the metals have residual positive ions. The electrostatic attraction between these positive ions and negative electrons is responsible for holding the solid together. This type of bonding is called metallic bonding.

PROPERTIES

The metallic bond is electrostatic in nature, though partially, and do not exert directional influence. The metallic bond is weaker in nature than the covalent bond because of the fewer electrons bonding the nuclei. But it can be stronger for those metals in which the number of valence electrons is greater. They are good conductors. Because of the presence of free electrons, they have high thermal and electrical conductivities.

HYDROGEN BOND:

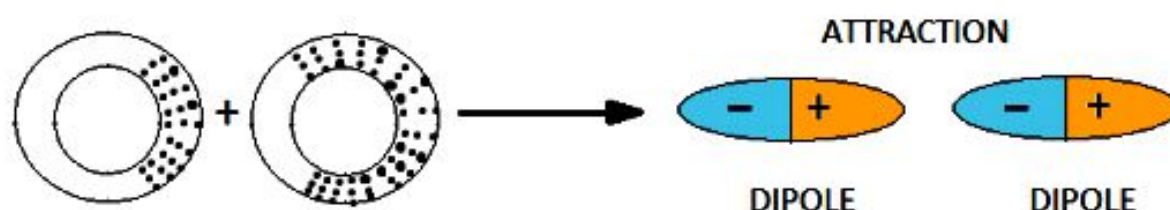
In certain crystals, a positive hydrogen ion (H^+) attracts negative ions such F^- , O^- , N^- , etc. sometimes due to electrostatic attraction, attachment between atoms in different molecules or within a molecule occurs in addition to bonds, which holds atoms together to form molecules. A hydrogen bond is formed when a hydrogen atom makes such an attachment or association with an electronegative atom like oxygen,



nitrogen, fluorine, etc. The hydrogen bond is formed in H_2O , HF and in many organic molecules. In the water molecule (H_2O), the hydrogen and oxygen atoms are held together by covalent bonds. The positive dipole end, i.e. hydrogen, can strongly attract the negative dipole end of water molecule.

VANDER WAAL BONDS OR MOLECULAR BOND:

Many solids or crystals are composed of neutral atoms or molecules without any transferring or sharing of the electrons. Hence the ionic covalent and metallic bonds are not possible between these atoms. These substances remain bound by much weaker short range attractive forces, which are called Vander Waals forces. These forces are weaker than the atomic bonding forces. The Vander Waals bonds are usually found in inert gases in which outermost electron orbits are completely filled, i.e., there are no valence electrons and hence they are incapable of forming any bond. These bonds are formed due to electrostatic attraction between oscillating or permanent dipoles.



In the case of inert gases, there is a very small attraction between the atoms due to closed outer shells. These gases condense when the temperature is reduced and hence a weak interatomic attraction is developed due to Vander Waals forces. The Vander Waals forces are non-directional in nature and a little energy is required to break the bonds because these are much weaker than ionic and covalent bonds. These types of bonds are found in inert gases like solid argon and in many organic molecules like methane.

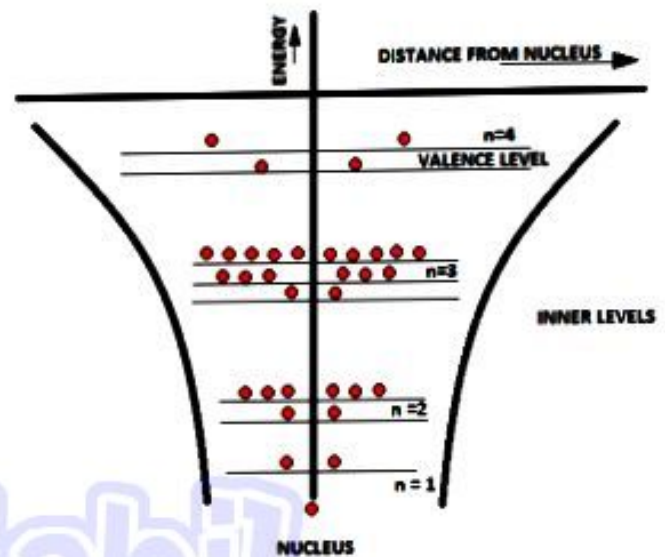
ENERGY BAND IN SOLIDS:

As we know the electron in the atom are distributed in different orbits i.e. different allowed energy levels. The electron first occupies that orbit in which its energy is minimum and later higher energy orbits are filled up. The electrons in the outermost orbit, which is fully or partially filled, are called valence electrons and that energy level is called valence level.

The valence electrons in an atom are at a larger distance from the nucleus compared to other electrons in the inner orbits. The attractive Colombian force on the valence electron is reduced considerably so that the valence electrons are responsible for the formation of bonds between the atoms.

In metals particularly electron which are moves freely are called 'free electrons' The other electrons in the atoms being in inner orbits are more strongly bound with the nucleus due to the electrostatic attractive force and cannot move out of the orbits. These electrons are called 'bound electrons'.

e.g. The germanium atom has 32 electrons which are distributed as $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^2, 4p^2$. The first three orbits are completely filled and the electrons in these orbits are bound electrons. The four electrons present in the fourth orbit are valence electrons.



In a single atom the electrons have a negative potential energy due to the attraction of the nucleus so that the electrons remains bound in the potential field of the nucleus. These electrons remain in the allowed energy states.

If two atoms are brought close to each other than the nucleus of one atom is in a positions to interact with the electrons of the other atoms. Due to this interaction each energy level of both the atoms splits into two levels. One of the levels is at a higher level than the earlier position while the other one is at a lower level. Further due to superposition the common wall of the potential well gets suppressed and the higher energy levels, even the valence levels become common for both the atoms. This results in a transfer of valence electrons from one atom to the other.

Since the interatomic distance is very small so that every atom is in a position to interact with a number of neighboring atoms, resulting in splitting of energy levels. Each energy level is now converted into a group of levels on both sides of the original level. The separation between these levels in the group is so small that they almost form a continuous band, called energy band.

The energy band in which the valence electrons are situated is called valence band. The allowed energy band above the valence band which is almost empty in the unexcited state is called conduction band. In between the conduction and valence bands there is an energy gap in which no allowed energy level is situated, this energy gap is called forbidden energy gap.

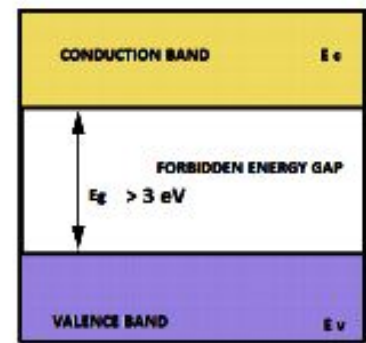


CLASSIFICATION OF SOLIDS ON THE BASIS OF ENERGY BAND GAP:

On the basis of energy band structure and electrical conductivity the solids can be divided into three types:

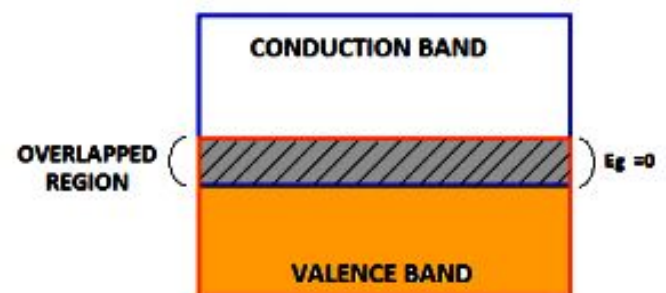
1. INSULATOR
2. CONDUCTOR
3. SEMICONDUCTOR

1. INSULATORS: For these types of solids, the band formation is like the forbidden gap is very wide, it is about **3eV to 6 eV**. It is seen that a very few electrons from the filled band reach the empty band, even if we thermally excite them or apply an electrical field to them. Due to this reason, a free electron current cannot be obtained and the solids of this type are poor conductors of electricity. This class of solids are known as insulators. The conductivity is very small of the order of 10^{-12} to 10^{-18} mho/m.



INSULATORS

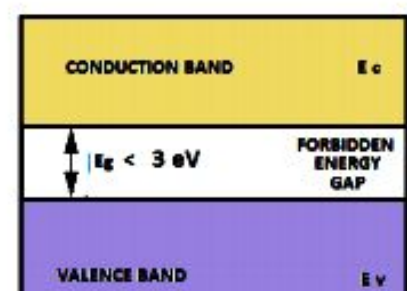
2. CONDUCTOR: In this kind of solids the valence and conduction bands are overlapped. Thus in these materials the forbidden energy gap is not present i.e. $\Delta E_g = 0$. Metals like silver copper etc. have an overlapping of bands while in sodium the valence band is only partially filled so that the valence band itself acts as conduction band. In conductors due to the availability of empty allowed states the electrons are capable of receiving energy from an applied field. As these electrons are very loosely bound with the nuclei, these get separated from the parent atoms and move almost freely. Due to the applied external field their random motion is affected and drift in a particular direction.



CONDUCTOR

Due to large number of free electrons the conductivity of such materials is very high or the resistivity is very low. At room temperature electrical conductivity is of the order of 10^6 to 10^8 mho/m.

3. SEMICONDUCTOR: For this type of solids the forbidden gap is very narrow of about **0.1eV to 1eV**. Under this situation electrons can move easily from the highest filled band (valence band) to the next empty band (conduction band) when applied external energy. This can be achieved by the thermal excitation or also by applying an electrical field.



SEMICONDUCTORS

For this reason, a free electron current can be obtained as a few electrons are available in the empty band. This class of solids is known as semiconductors. Silicon and Germanium are the examples of semiconductors.

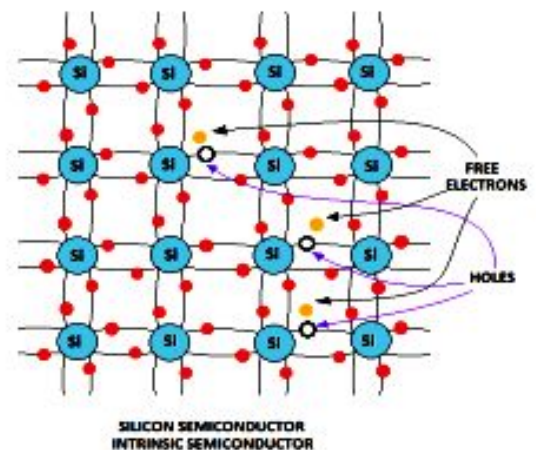
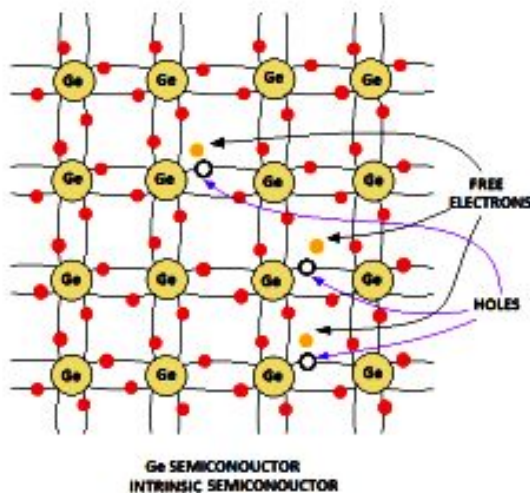
There also exists another electric current which is due to empty places left behind when the electrons moves, which remain near the top of the uppermost filled band. These vacancies (empty space) are called holes. The holes behave as positive electrons and can contribute to the generation of electric current. This is possible as the electron below the hole may gain enough energy to jump and occupy the hole due to the applied electric field. With such successive jumps of electrons, the hole moves towards the lower energy state and contribute to the generation of electric current.

There are two types of semiconductors

(A) INTRINSIC SEMICONDUCTORS

(B) EXTRINSIC SEMICONDUCTORS

(A) INTRINSIC SEMICONDUCTORS: The pure and natural form of semiconductors are known as intrinsic semiconductors. The transformation of electrons to the conduction band and the generation of holes in the valence band are achieved by thermal excitation. It means this effect is temperature dependent and produces equal numbers of electron and hole carriers.



CONDUCTIVITY IN INTRINSIC SEMICONDUCTOR: At absolute zero temperature all the valence electrons remain tightly bound in the covalent bonds so that there is no electron in the conduction band and the conductivity of the material is zero. Thus all semiconductors at absolute zero behave as insulators. As the temperature increases some electrons gain sufficient thermal energy and become free from bonding (mutual breaking of covalent bonds). As soon as

an electron becomes free after breaking the bond it leaves behind a vacant site. This vacancy or vacant site is called a **hole**.

If in an intrinsic semiconductor the number of free electrons and holes per unit volume are n_i and p_i then

$$n_i = p_i$$

$$n_i \times p_i = n_i^2 \quad \text{-----(1)}$$

In an intrinsic semiconductor at temperature T K

$$n_i = A T^{3/2} \exp(-\Delta E_g / 2kT) \quad \text{-----(2)}$$

Where A is constant and k is Boltzmann constant. It decreases exponentially with increase of forbidden energy gap

At normal temperature the free electrons in the conduction band have a random motion like the molecules of gas and the holes in the valence band also move randomly.

Suppose in a crystal lattice of intrinsic semiconductor an electron in covalent bond becomes free by the external thermal energy. This creates a hole at that place. This electron with some thermal energy breaks another covalent bond of neighboring atom and new freed electron acquires the position of hole at previous position so that the covalent bond is completed but a hole is created at new position. In the same way if a transition of electron takes place from one atom to another and so on. Thus the random motion of an electron on the path results in the random motion. Both free electrons in conduction band and holes in valence band act as charge carriers and the electrical conduction is due to both types of charge carriers in semiconductors.

When a battery is connected between the two ends of semiconductor then due to the electric field the free electrons experience a force towards that positive terminal of battery and holes towards the negative terminal. The motion of these charges carriers in a definite direction is called drift velocity. So

The drift velocity of electrons

$$v_e = \mu_e E \quad \text{-----(1)}$$

The drift velocity of holes

$$v_h = \mu_h E \quad \text{-----(2)}$$

Here μ_e = Mobility of electrons

μ_h = Mobility of holes

Since the total current density in an intrinsic semiconductor is

$$J = J_e + J_h \quad \text{-----(3)}$$

$$\text{But } J_e = n (-e)(-v_e) = \mu_e n e E$$

$$\text{And } J_h = p (+e)(+v_h) = \mu_h p e E$$

$$J = e E (\mu_e n + \mu_h p)$$

$$\text{And CONDUCTIVITY } \sigma = \frac{J}{E} = e \frac{E(\mu_e n + \mu_h p)}{E} = e (\mu_e n + \mu_h p)$$

$$\text{And RESISTIVITY } \rho = \frac{1}{\sigma} = \frac{1}{e (\mu_e n + \mu_h p)}$$

EXTRINSIC SEMICONDUCTORS: As we know conductivity of intrinsic semiconductors are very low. To enhance the conductivity intrinsic semiconductors are doped with a controlled amount of an impurity elements. The obtained semiconductors are so called as EXTRINSIC SEMICONDUCTORS. The impurity elements are taken from 3rd and 5th group of periodic table.

There are two types of extrinsic semiconductors

1. N- type semiconductors
2. P- type semiconductors

N- TYPE SEMICONDUCTORS: If an element of 5th group (P, As, Sb) is added to pure semiconductor, the obtained semiconductor is called as N-type semiconductor. These added atoms of 5th group having 5 electrons in the valence orbit (Pentavalent impurity). These impurities are known as **DOPANT** and the process by which these dopant are added to intrinsic semiconductor is known as **DOPING**. In doping process these dopants replace an atom of intrinsic semiconductor and 4 out of 5 electron forms covalent bonds but one electron remains free. At room temperature this free electron is transferred to the conduction band.

The energy level of the valence electron of the pentavalent impurity element lies in the forbidden energy gap of the intrinsic semiconductor just below the conduction band. This energy level is called donor level. The electrons in the donor level can be easily transferred to the conduction band. Since there are negatively charge carrier are taking part in flow of current so the extrinsic semiconductor is called as N-type semiconductor.

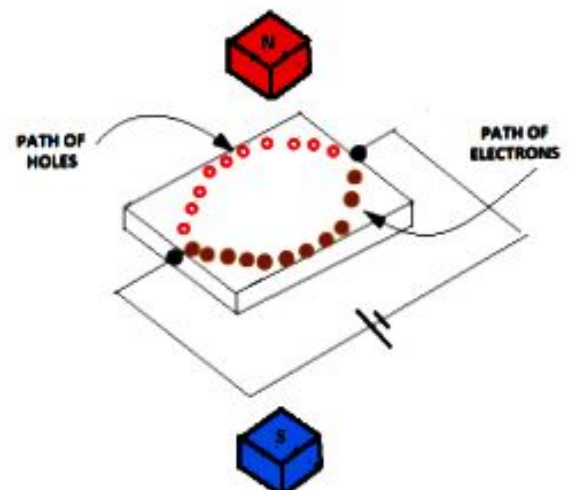
P- TYPE SEMICONDUCTORS: When an element of 3rd group (B, In, Ga) is added as impurity in pure semiconductor, the obtained semiconductor is called P-type semiconductor. Since the 3rd group elements are trivalent. In doping process these dopants replace an atom of

intrinsic semiconductor and all 3 electrons forms covalent bonding with neighboring pure semiconductor atom but 4th bond is mutually formed with pure semiconductor atom. Due to this mutual bond a deficiency of electron exists there. This deficiency or vacant site of an electron is called **HOLE** and it has the capability of capturing an available electron. The impurity of third group is called **ACCEPTOR IMPURITY**.

The energy level of the hole produced by the addition of 3rd group impurities lies in the forbidden energy gap slightly above the valence band of the semiconductor. This energy gap slightly above the valence band of the semiconductor.

Each acceptor impurity atom generates one hole which acts as a positively charged carrier. Thus a semiconductor doped with trivalent impurity has holes as majority charge carriers. As the electrical conduction now is mainly by the holes which behave as positively charged carriers, the doped semiconductor is called **P-TYPE SEMICONDUCTOR**.

HALL EFFECT: When a semiconducting (or conducting) plate in which current is flowing is placed in a magnetic field perpendicular to the direction of flow of current then a potential difference appears between the opposite faces of the plate i.e. a transverse electric field is developed perpendicular to direction of current and magnetic field. This effect is called as **HALL EFFECT**.



THEORY: Consider a semiconductor bar placed along x-z plane in which an electric current I is flowing in the x- direction by using a battery. A magnetic field $\vec{B}$ is applied in the z- direction.

Due to the potential due to battery charge carriers (electrons) drifted in the $-x$ direction while the positive charge carriers (ions or holes) will drift in $+x$ direction.

Let q be the charge on carriers and $\vec{v}$ be their average drift velocity. Due to the external magnetic field $\vec{B}$ a force will act on the moving charge carriers. This force will be:

$$\vec{F} = q (\vec{v} \times \vec{B})$$

For electrons $q = -e$ $\vec{v} = v(-\hat{x})$ and $\vec{B} = B(\hat{z})$

$$\vec{F} = -e (v(-\hat{x}) \times B(\hat{z}))$$

$$\vec{F} = e v B (-\hat{y}) \quad \text{i.e. the electrons will suffer a force in the direction of } y\text{-axis.}$$

$$\text{For holes} \quad q = +e \quad \vec{v} = v(\hat{x}) \quad \text{and} \quad \vec{B} = B(\hat{z})$$

$$\vec{F} = e (v(\hat{x}) \times B(\hat{z}))$$

$$\vec{F} = e v B (-\hat{y}) \quad \text{i.e. holes will also suffer a force in the direction of } y\text{-axis.}$$

With electrons as a charge carriers the electrons will be collected at the bottom of the semiconductor bar and the top of the bar will become positively charged due to lack of electrons. As a result a transverse electric field $\hat{E}_t$ called HALL ELECTRIC FIELD will be produced which will oppose the downward movement of electrons.

In the same way with positive charge carriers, they will also be deflected downward and will collect on the bottom of the semiconductor bar. But now the transverse electric field $\hat{E}_t$ will be upward (+ y direction) opposing the downward movement of positive charge carriers.

A steady state will be achieved when the force due to magnetic field will be balanced by the force due to transverse electric field (Hall field).

In steady state

$q (\vec{v} \times \vec{B}) = -q \hat{E}_t \quad (\text{FOR ELECTRONS})$ $\hat{E}_t = (\vec{v} \times \vec{B})$ $\hat{E}_t = -v B (-\hat{x} \times \hat{z})$ $\hat{E}_t = v B (-\hat{y})$	$q (-\vec{v} \times \vec{B}) = q \hat{E}_t \quad (\text{FOR HOLES})$ $\hat{E}_t = -(\vec{v} \times \vec{B})$ $\hat{E}_t = -v B (-\hat{x} \times \hat{z})$ $\hat{E}_t = v B (+\hat{y})$
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HALL COEFFICIENT:

The average drift velocity $\vec{v}$ is related to the current density $\vec{J}$ as

$$\vec{J} = n q \vec{v} \quad \therefore$$

$$v = \frac{J}{n q}$$

Since $E_t = v B \quad \therefore \quad E_t = \frac{J}{n q} (B) = \frac{1}{n q} (J B)$

If the thickness of the semiconductor bar is 'd'

Then $E_t = \frac{V_h}{d}$

$$\frac{V_h}{d} = \frac{1}{n q} (J B)$$

or $V_h = \frac{d}{n q} (J B)$

is known as **HALL VOLTAGE**

And the factor $\frac{1}{n q} = R_h$

is known as **HALL COEFFICIENT.**

SYLLABUS

UNIT 6

ELECTRODYNAMICS

- ✓ INTRODUCTION
- ✓ SCALAR FIELDS
- ✓ VECTOR FIELD
- ✓ DEL OPERATOR
- ✓ GRADIENT
 - GRADIENT OF SCALAR IN CARTESIAN COORDINATES
 - PHYSICAL SIGNIFICANCE OF GRADIENT:
- ✓ DIVERGENCE
 - DIVERGENCE OF A VECTOR FIELD:
 - PHYSICAL SIGNIFICANCE OF DIVERGENCE OF A VECTOR FIELD
- ✓ CURL
 - PHYSICAL SIGNIFICANCE OF CURL
- ✓ LAPLACE AND POISSON'S EQUATION
- ✓ BIOT –SAVART'S LAW
 - MAGNETIC FIELD IN VECTOR FORM
 - DIVERGENCE OF MAGNETIC FIELD
 - CURL OF MAGNETIC FIELD
- ✓ FARADAY'S LAW
- ✓ DISPLACEMENT CURRENT
- ✓ MAXWELL'S EQUATIONS
 - MAXWELL'S FIRST EQUATION (Gauss's law for electric field)
 - MAXWELL'S 2ND EQUATION (Gauss's law for magnetic field)
 - MAXWELL'S 3RD EQUATION (Faraday's law for EMI)
 - MAXWELL'S FOURTH EQUATION (Ampere's circuital law)
- ✓ POYNTING VECTOR AND POYTING THEOREM

INTRODUCTION

As we know separation of charges produces electric field, whereas the motion of charges generates current and hence the magnetic field. When these fields are time varying they are coupled with each other through Maxwell's equations. With the Maxwell's equations we can derive wave equation, based on which the propagation of electromagnetic waves can be investigated in different medium.

Electromagnetism is one the most important interaction occurring in nature. From the engineering point of view we can hardly find an area where the effect of electromagnetism is not observed. This chapter gives mathematical skills to understand electromagnetism.

FIELDS: The field or space, in which the physical quantity depends upon the coordinates of the point of the observation is called field of that physical quantity.

According to the nature of physical quantity, fields are of two types:

1. Scalar field
2. Vector field

SCALAR FIELDS:

That field or space in which a scalar quantity depends upon the coordinates or the point of observation, is called a scalar field.

If a scalar quantity at a point having coordinates x, y and z is ϕ then

$$\phi = \phi(x, y, z)$$

Physical quantities such as electric potential, gravitational potential, temperature, density etc. are expressed in the form of scalar fields. Scalar quantities are graphically represented by drawing surfaces at which the value of the scalar quantity remains a constant. Since one point in space corresponds to only one value of the scalar quantity i.e., scalar fields are single valued functions, hence these surfaces, called equiscalar surfaces and do not intersect with each other.

For example : If we put a point charge in any place, then the electric potential around it will depend on the position of the point. Since electric potential is a scalar quantity, the field around the charge will be known as scalar potential field. If all such points at which the potential is constant are joined by a surface then such a surface is called equipotential surface.

VECTOR FIELD:

If the magnitude or direction or both of a vector quantity change with the change in position of the point of observation in a field, that is the vector quantity depends on the coordinates of the point of observation, the field is called vector field.

e.g. $\vec{A} = \vec{A}(x, y, z)$

Vector fields are graphically represented by lines. These lines are called field lines or flux lines. The tangent at any point of the line gives the direction of the vector field at that point. The magnitude of a vector field at any point of the field is equal to the number of field lines going out of unit area taken normal to the direction of the vector at that point.

Suppose the number of field lines, or flux going out of an area element $\vec{da}$ perpendicular to the field lines at any point is $d\phi$ then the magnitude of the intensity of the vector field will be:

$$|\vec{A}| = \frac{d\phi}{da}$$

$\therefore$ vector field $|\vec{A}| = \left(\frac{d\phi}{da}\right) \hat{n}$

Where $\hat{n}$ is the unit vector perpendicular to the area (da)

$\therefore \hat{n} da = \vec{da}$

Taking the dot product of unit vector $\hat{n}$ with the vector field $\vec{A}$

$$\vec{A} \cdot \hat{n} = \frac{d\phi}{da} \hat{n} \cdot \hat{n} = \frac{d\phi}{da}$$

Or $d\phi = (\vec{A} \cdot \hat{n}) da = \vec{A} \cdot \hat{n} da$

$$\text{Flux} = d\phi = \vec{A} \cdot \vec{da}$$

DEL OPERATOR:

The del operator is the differential operator, which is represented by $\vec{\nabla}$ and is given by $\vec{\nabla} =$

$$\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

e.g. $\vec{\nabla} F = \hat{i} \frac{\partial F}{\partial x} + \hat{j} \frac{\partial F}{\partial y} + \hat{k} \frac{\partial F}{\partial z}$

The del operator can act in different ways

When del operator acts on a scalar function F , the resultant $\vec{\nabla}F$ is called the GRADIENT OF SCALAR FUNCTION.

When it acts on a vector function $\vec{A}$ via the dot product, the resultant is $\vec{\nabla} \cdot \vec{A}$ is called the divergence of $\vec{A}$.

When it acts on a vector function $\vec{A}$ via the cross product, the resultant $\vec{\nabla} \times \vec{A}$ is called the curl of a vector $\vec{A}$.

GRADIENT:

If we think of the derivative of a function of one variable, gradient tells us how fast the function varies if we move a small distance. It means the gradient is the rate of change of a quantity with distance.

Suppose in a scalar field two surfaces with constant scalar quantities ϕ and $\phi + d\phi$ respectively. The position vector of any point A on S_1 relative to the origin O is $(\vec{r})$ and the position vector of another point B on S_2 is $(\vec{r} + \delta\vec{r})$

Displacement of B with respect to A is

$$\overrightarrow{AB} = \delta\vec{r}$$

And the rate of change of the scalar function in the direction of AB = $\frac{\delta\phi}{\delta r}$ -----(1)

Now we draw a normal AC from A on surface S_2 .

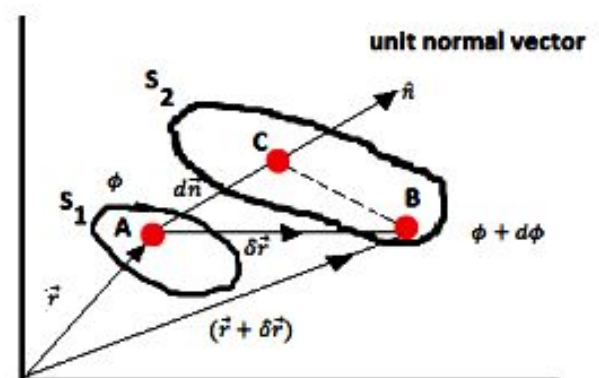
The minimum distance $AC = \delta n$ between the surfaces S_1 and S_2 will be in the direction of their normal

The rate of change of scalar function ϕ in the direction of AC = $\frac{\delta\phi}{\delta n}$

If the distance δr is very small then $\triangle ABC$ can be right angle triangle. Suppose the angle between $\delta\vec{r}$ and $d\vec{n}$ is θ then,

$$AC = AB \cos \theta$$

$$\delta n = \delta r \cos \theta$$



Or $\delta r = \frac{\delta n}{\cos \theta}$ putting this in equation (1)

The rate of change of the scalar function in the direction of AB = $\frac{\delta \phi}{\delta n} \cos \theta$

The maximum value of $\cos \theta$ is 1

So the maximum rate of change of $\phi = \left(\frac{\delta \phi}{\delta r}\right)_{max} = \left(\frac{\delta \phi}{\delta n}\right)$

In the limit $\delta r \rightarrow 0$ the maximum rate of change of $\phi = \left(\frac{d\phi}{dr}\right)_{max} = \left(\frac{d\phi}{dn}\right)$

So the rate of change of a scalar function is maximum in a direction normal to the equiscalar surface.

In a scalar field, the maximum rate of change of the scalar function in space is known as the gradient of the scalar field.

It is a vector quantity and its direction is normal to the equiscalar surface i.e. the direction in which rate of change is maximum.

$$\text{gradient } \phi = \text{grad } \phi = \left(\frac{d\phi}{dn}\right) \hat{n} = \left(\frac{d\phi}{dr}\right)_{max} \hat{n}$$

GRADIENT OF SCALAR IN CARTESIAN COORDINATES

Since we know the definition of gradient:

$$\text{grad } \phi = \left(\frac{d\phi}{dn}\right) \hat{n} \quad \text{-----(1)}$$

Let a displacement vector

$$d\vec{r} = i dx + j dy + k dz \quad \text{-----(2)}$$

taking the dot product of equation 1 and 2

$$(\text{grad } \phi) \cdot d\vec{r} = \left(\frac{d\phi}{dn}\right) \hat{n} \cdot d\vec{r}$$

Here angle between $d\vec{r}$ and $\hat{n}$ is θ therefore

$$\hat{n} \cdot d\vec{r} = dr \cos \theta = dn \quad \because \frac{dn}{dr} = \cos \theta$$

$$(\text{grad } \phi) \cdot d\vec{r} = \left(\frac{d\phi}{dn}\right) dn$$

$$(\text{grad } \phi) \cdot d\vec{r} = d\phi \quad \text{-----(3)}$$

Now since we know

$$d\phi = \left(\frac{\partial\phi}{\partial x}\right)dx + \left(\frac{\partial\phi}{\partial y}\right)dy + \left(\frac{\partial\phi}{\partial z}\right)dz \quad \text{-----(4)}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z) \cdot (\hat{i} B_x + \hat{j} B_y + \hat{k} B_z)$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad \text{Comparing with equation 4}$$

$$d\phi = \left(\hat{i} \frac{\partial\phi}{\partial x} + \hat{j} \frac{\partial\phi}{\partial y} + \hat{k} \frac{\partial\phi}{\partial z}\right) \cdot (\hat{i} dx + \hat{j} dy + \hat{k} dz)$$

$$d\phi = \left(\hat{i} \frac{\partial\phi}{\partial x} + \hat{j} \frac{\partial\phi}{\partial y} + \hat{k} \frac{\partial\phi}{\partial z}\right) \cdot d\vec{r} \quad \text{-----(5)}$$

From equation 3 and 5

$$\text{grad } \phi = \left(\hat{i} \frac{\partial\phi}{\partial x} + \hat{j} \frac{\partial\phi}{\partial y} + \hat{k} \frac{\partial\phi}{\partial z}\right)$$

Or

$$\nabla\phi = \left(\hat{i} \frac{\partial\phi}{\partial x} + \hat{j} \frac{\partial\phi}{\partial y} + \hat{k} \frac{\partial\phi}{\partial z}\right) = \text{grad } \phi$$

PHYSICAL SIGNIFICANCE OF GRADIENT:

DIVERGENCE OF A VECTOR FIELD:

The vector flux per unit volume emerging out of the closed surface is enclosing a volume element placed in a vector field is known as the divergence of a vector field.

Suppose a closed surface enclosing a volume dV is situated in vector field $\vec{A}(x, y, z)$. The flux emerging out of the closed surface will be:

$$\phi = \oint_S \vec{A} \cdot d\vec{a}$$

Therefore, the divergence of the vector $\vec{A}$ at the position of the element will be defined as

$$\text{div } \vec{A} = \lim_{dV \rightarrow 0} \left[\frac{1}{dV} \oint_S \vec{A} \cdot d\vec{a} \right] = \vec{\nabla} \cdot \vec{A}$$

Here $\oint_S$ represents integration over a closed surface. The divergence of a vector field is a scalar quantity.

As explained earlier

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A}$$

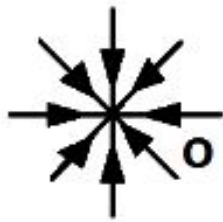
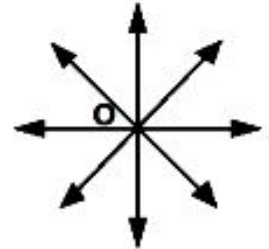
$$\text{div } \vec{A} = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}\right) \cdot (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z)$$

$$\text{div } \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergence of vector field may be positive, negative or zero.

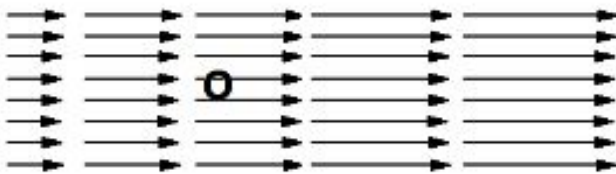
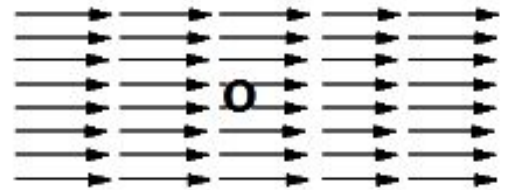
PHYSICAL SIGNIFICANCE OF DIVERGENCE OF A VECTOR FIELD:

To understand the physical significance or physical meaning of divergence of a vector field, at point O the divergence of vector field is +ve as the vector spread out



As it is clear from fig. that the vector converges and so the divergence at point O is negative.

As shown in fig. it is clear that the divergence of a vector field is zero as the magnitude of vector remains same.



As shown in fig. the magnitude of the vectors get increased as move towards right. Hence the divergence of such vector field is not zero but positive.

CURL:

Curl of a vector field is defined as the cross product of $\vec{\nabla}$ and $\vec{A}$

i.e. $\text{Curl of } \vec{A} = \vec{\nabla} \times \vec{A}$

$$\text{curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\text{Curl } \vec{A} = \hat{i} \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) + \hat{j} \left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \right) + \hat{k} \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right)$$

Clearly we can say that curl is a vector quantity, i.e. it has magnitude and direction both. Also we can say that the curl of scalar field is meaningless since cross product of scalar with vector $\vec{\nabla}$ is meaningless.

So curl of any vector field is defined as the line integral per unit infinity small area enclosed by the path.

$$\text{Curl } \vec{A} = \lim_{ds \rightarrow 0} \left(\frac{\oint \vec{A} d\vec{l}}{ds} \right) \hat{n}$$

The direction of curl can be determined by the right hand thumb rule i.e. perpendicular to the plane in which line integral along the surface element is maximum.

PHYSICAL SIGNIFICANCE OF CURL:

The term curl is a measure of how much a vector field $\vec{A}$ curls around the point of observation. Curl of any vector field comes from its rotational motion.

To understand this let us assume flow of water in some water body as shown in fig.

At point P where $\vec{A}$ (Area vector) is same at all positions, we put a wooden block and look for its motion. Wooden block will show linear motion and doesn't show any rotational motion, so we can say curl at P will be zero. For vector $\vec{v}$ (previously took $\vec{A}$)

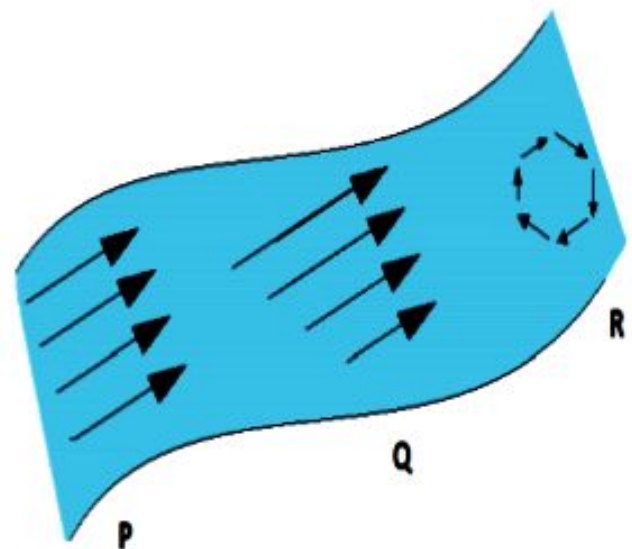
$$\text{Curl } \vec{v} = 0$$

While putting the same wooden block at point Q wooden block starts rotational motion in clockwise direction since area field vectors are not equal, so the

$$\text{Curl } \vec{v} \neq 0$$

Similarly at point R wooden block shows maximum rotational motion in clockwise direction so can say

$$\text{Curl } \vec{v} = \text{maximum}$$



LAPLACE AND POISSON'S EQUATION:

Since we know the electric field $\vec{E}$ in a region using Coulomb's Law or Gauss's Law when the charge distribution is specified in the region for $\vec{E} = -\nabla V$. When the potential V is specified throughout the region.

However in practical cases neither the charge distribution nor the potential distribution is specified at some boundaries. These types of problems are known as Boundary value problems. For these problems the field and the potential are determined by using Poisson's equation or Laplace equation.

∴ Gauss's equation $\nabla \cdot \vec{D} = \rho v$ Here $\vec{D}$ = Flux density

$$\lim_{v \rightarrow 0} \frac{\oint \vec{D} \cdot d\vec{s}}{v} = \frac{Q}{v}$$

$$\lim_{v \rightarrow 0} \frac{\oint \vec{D} \cdot d\vec{s}}{v} = \rho v$$

$$\nabla \cdot \vec{D} = \rho v$$

But $\vec{D} = \epsilon \vec{E}$

ρ = Volume charge density

$\therefore \nabla \cdot (\epsilon \vec{E}) = \rho v$

v = Volume

but $\vec{E} = -\nabla V$

Here V = Electric Potential

$\nabla \cdot (-\epsilon \nabla V) = \rho v$

$-\epsilon (\nabla \cdot \nabla V) = \rho v$

$\nabla^2 V = \rho \frac{v}{\epsilon}$ i.e. known as Poisson's Equation.

In certain region where $\rho v = 0$ (For dielectric medium)

Then $\nabla^2 V = 0$ i.e. Laplace Equation.

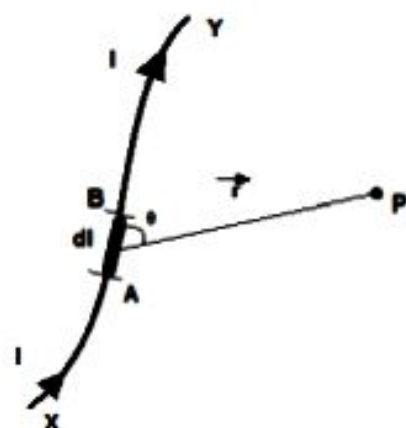
BIOT –SAVART'S LAW:

This law deals with the magnetic field at a point due to a small current element. (A part of a conductor carrying current).

Let us consider a small element AB of the conductor XY carrying current I . Let $\vec{r}$ be the position vector of point P from the current element $I \cdot d\vec{l}$ and θ be the angle between $d\vec{l}$ and $\vec{r}$.

According to Biot- Savarts law magnitude of magnetic field dB at a point P due to current element depends upon :-

- | | | |
|-------|-------------------|--------------------------|
| (i) | Current | $dB \propto I$ |
| (ii) | Element length | $dB \propto dl$ |
| (iii) | Sine of angle | $dB \propto \sin \theta$ |
| (iv) | Distance of point | $dB \propto 1 / r^2$ |



Combining these factors

$dB \propto \frac{I \, dl \, \sin \theta}{r^2}$

or $dB = k \frac{I \, dl \, \sin \theta}{r^2}$

Here k = proportionality constant

$K = \frac{\mu_0}{4\pi}$

μ_0 = Magnetic permeability of free space

$\mu_0 = 4\pi \times 10^{-7} \text{ Wb} \cdot \text{A}^{-1} \text{ m}^{-1}$

$\mu_0 = 4\pi \times 10^{-7} \text{ Tesla A}^{-1} \text{ m}.$

MAGNETIC FIELD IN VECTOR FORM :-

$$\vec{dB} = k \frac{I (\vec{dl} \times \vec{r})}{r^3}$$

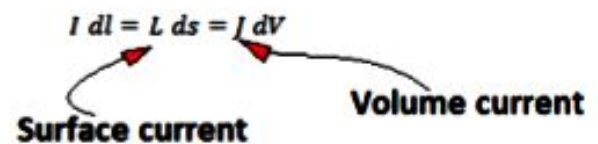
$$\text{Or } \vec{B} = \frac{1}{4\pi} \int_L \frac{I d\vec{l} \times \vec{a}_r}{r^2} \text{ (Line integral)}$$

$$k = \frac{1}{4\mu} \quad \text{here } \mu = 1 \text{ (let)}$$

Similarly:

$$\vec{B} = \frac{1}{4\pi} \int_S \frac{L d\vec{s} \times \vec{a}_r}{r^2} \text{ (Surface integral)}$$

$$\vec{B} = \frac{1}{4\pi} \int_V \frac{J d\vec{V} \times \vec{a}_r}{r^2} \text{ (Volume integral)}$$



DIVERGENCE OF MAGNETIC FIELD:

To find the divergence of magnetic field let us consider biot savart's law in the form of volume integral.

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V (\vec{J} \times \frac{\vec{r}}{r^3}) dV$$

Taking divergence both side

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int_V \vec{\nabla} \cdot (\vec{J} \times \frac{\vec{r}}{r^3}) dV$$

Now using vector triple product

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot (\vec{a} \times \vec{b}) - \vec{b} \cdot (\vec{a} \times \vec{c})$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \oint_V \left[\frac{\vec{r}}{r^3} \cdot (\vec{\nabla} \times \vec{J}) - \vec{J} \cdot (\vec{\nabla} \times \frac{\vec{r}}{r^3}) \right] dV$$

$$\vec{\nabla} \times \vec{J} = 0$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \oint_V \left[\frac{\vec{r}}{r^3} \cdot (0) - \vec{J} \cdot (\vec{\nabla} \times \frac{\vec{r}}{r^3}) \right] dV$$

$$\vec{\nabla} \cdot \vec{B} = - \frac{\mu_0}{4\pi} \oint_V \left[\vec{J} \cdot (\vec{\nabla} \times r^{-3} \vec{r}) \right] dV$$

$$\text{Again using } \vec{\nabla} \times \phi \vec{a} = \vec{\nabla}(\phi) \times \vec{a} + \phi (\vec{\nabla} \times \vec{a}) \quad \text{(Mathematical formula)}$$

$$\vec{\nabla} \cdot \vec{B} = - \frac{\mu_0}{4\pi} \oint_V \left[\vec{J} \cdot (\vec{\nabla}(r^{-3}) \times \vec{r}) + r^{-3} (\vec{\nabla} \times \vec{r}) \right] dV$$

$$\vec{\nabla} \cdot \vec{B} = - \frac{\mu_0}{4\pi} \oint_V [\vec{j} \cdot ((-3r^{-3-2} \vec{r}) \times \vec{r}) + r^{-3} (0)] dV$$

$$\vec{\nabla} \cdot \vec{B} = - \frac{\mu_0}{4\pi} \oint_V [\vec{j} \cdot (-3r^{-3-2} (\vec{r} \times \vec{r})) + 0] dV$$

$$\vec{\nabla} \cdot \vec{B} = - \frac{\mu_0}{4\pi} \oint_V [\vec{j} \cdot 0] dV$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

CURL OF MAGNETIC FIELD:

From Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

From stocke's theorem:

$$\oint (\text{curl } \vec{B}) \cdot d\vec{s} = \oint \vec{B} \cdot d\vec{l} \quad \text{and} \quad J = \frac{I}{s} \quad \text{or we can say } I = \oint \vec{j} \cdot d\vec{s}$$

$$\oint (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \oint \vec{j} \cdot d\vec{s}$$

$$(\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \vec{j} \cdot d\vec{s}$$

$$(\vec{\nabla} \times \vec{B}) = \mu_0 \vec{j}$$

FARADAY'S LAW: According to Faraday's law of induction that an EMF is induced in the closed loop when the number of magnetic field lines changes (or there is change in flux).

$$\text{Since we know} \quad e = - \frac{\partial \phi}{\partial t} \quad \text{-----(1)}$$

$$\text{And} \quad \phi = \oint \vec{B} \cdot d\vec{s} \quad \text{also} \quad e = \oint_L \vec{E} \cdot d\vec{l}$$

Putting these values in equation (1)

$$\oint_L \vec{E} \cdot d\vec{l} = - \frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s}$$

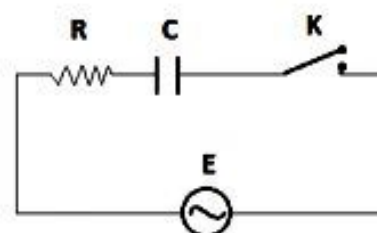
$$\text{Applying stokes theorem i.e.} \quad \oint_L \vec{A} \cdot d\vec{l} = \oint_S (\text{curl } \vec{A}) \cdot d\vec{s}$$

$$\oint_S (\text{curl } \vec{E}) \cdot d\vec{s} = - \frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s}$$

$$\text{So we can say} \quad \text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{i.e Faraday's Law for electromagnetism.}$$

DISPLACEMENT CURRENT:

As per experiment conducted by Maxwell the magnetic fields are generated not only due to flow of charge(current) but also due to time dependent changes in the electric field. So this type of time varying field is equivalent to a current called displacement current. To understand the concept of displacement current, consider a parallel plate capacitor connected to a battery as shown:



When key is pressed, the current changes in the circuit from zero to its peak value exponentially similarly when key is released the current drops to zero from its maximum value. As we know that for a flow of current, circuit should be continuous but in this case current should not flow in a capacitor because of the gap between the plates, but in reality the current does flow. For explaining this Maxwell put the idea of displacement current flowing in the vacuum between the two plates.

To calculate the current flowing between two plates of the capacitor since the electric field between the plates

$$\vec{E} = \frac{\sigma}{\epsilon_0}$$

here: $\sigma = \text{charge density}$

$$\text{And } D = \epsilon_0 \vec{E} = \vec{\sigma} \quad \text{-----(1)}$$

$D = \text{Displacement vector field}$

Since the rate of change of charge is equivalent to a current flowing in circuit when the capacitor is charged or discharged.

$$\text{Current} = I = \frac{\partial q}{\partial t}$$

Dividing by the area of the capacitor plate

$$\frac{I}{A} = \frac{\partial}{\partial t} \left(\frac{q}{A} \right)$$

Current density

$$J = \frac{I}{A}$$

And charge density

$$\sigma = \frac{q}{S}$$

Current density

$$\vec{J} = \frac{\partial}{\partial t} (\vec{\sigma})$$

From equation 1

$$\vec{J} = \frac{\partial}{\partial t} (\vec{D}) = \vec{J}_d$$

So we can say the rate of change of displacement vector is equivalent to current density. It is called displacement current density. It is equal to the density of current flowing in the outer circuit.

MAXWELL'S EQUATIONS:

Maxwell's equations describe the electric and magnetic field due to distribution of charges and current i.e. how these fields change in time.

These equations are differential and integral form of

- (i) Gauss's law for electric field
- (ii) Gauss's law for magnetic field
- (iii) Faraday's law for EMI
- (iv) Ampere's circuital law

1. MAXWELL'S FIRST EQUATION:

As we know Gauss's law for electric field states that the total electric flux passing through a closed surface is equal to the ratio of total charge and permittivity of medium.

$$\phi = \frac{Q}{\epsilon_0}$$

But we know $\phi = \oint \vec{E} \cdot d\vec{a}$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

If volume charge density is ρ then

$$Q = \oint_V \rho \, dV$$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{\oint_V \rho \, dV}{\epsilon_0}$$

This is integral form of Maxwell's 1st equation.

Since we know divergence theorem $\oint_S \vec{E} \cdot d\vec{a} = \oint_V (\vec{\nabla} \cdot \vec{E}) \, dV$

Therefore $\oint_V (\vec{\nabla} \cdot \vec{E}) \, dV = \frac{\oint_V \rho \, dV}{\epsilon_0}$

Or $(\vec{\nabla} \cdot \vec{E}) = \frac{\rho}{\epsilon_0}$

This is differential form of Maxwell's 1st equation.

Since $(\vec{\nabla} \cdot \vec{E}) = \frac{\rho}{\epsilon}$

Here $\epsilon = \text{Permittivity of medium}$

Or we can say $\vec{\nabla} \cdot \vec{E} \epsilon = \rho$

Or $\vec{\nabla} \cdot \vec{D} = \rho$

Here $\vec{D} = \text{Displacement field} = \vec{E} \epsilon$

So we can say $\oint_V \vec{D} \, dV = \rho$

MAXWELL'S 2ND EQUATION: Maxwell's 2nd equation is based on Gauss's law for magnetic field. According to this law, the number of magnetic field lines entering any surface is exactly the same as leaving it. It means the flux of magnetic field $\vec{B}$ across any closed surface is always zero. i.e.

$$\oint_S \vec{B} \cdot d\vec{a} = 0$$

Using divergence theorem:

$$\oint_S \vec{B} \cdot d\vec{a} = \oint_V (\text{div } \vec{B}) dV$$

$$\oint_V (\nabla \cdot \vec{B}) dV = 0$$

$\nabla \cdot \vec{B} = 0$ This is differential form of Maxwell's 2nd equation.

MAXWELL'S 3RD EQUATION: Maxwell's 3rd equation is based on Faraday's law of induction. i.e.

$$e = - \frac{\partial \phi}{\partial t}$$

$$\text{And } e = \oint_C \vec{E} \cdot d\vec{r}$$

$$\text{And } \phi = \oint_S \vec{B} \cdot d\vec{a}$$

$$\oint_C \vec{E} \cdot d\vec{r} = - \frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{a}$$

$\oint_C \vec{E} \cdot d\vec{r} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$ This is Integral form of Maxwell's 3rd equation.

Applying curl theorem:

$$\oint_C \vec{E} \cdot d\vec{r} = \oint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a}$$

$$\oint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$(\vec{\nabla} \times \vec{E}) = \frac{\partial \vec{B}}{\partial t}$$

Or **$\text{curl } \vec{E} = \frac{\partial \vec{B}}{\partial t}$** This is differential form of Maxwell's 3rd equation.

MAXWELL'S FOURTH EQUATION:

Maxwell's 4th equation is based on Ampere's circuital law. According to Ampere's law "Line integral of magnetic field over a closed path is equal to the product of μ and total current passes through the closed loop. i.e.

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \Sigma I \quad \text{-----(1)}$$

Since the current density is J then

$$\Sigma I = \oint_S \vec{J} \cdot d\vec{a}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \oint_S \vec{J} \cdot d\vec{a} \quad \text{-----(2)}$$

From curl theorem:

$$\oint_C \vec{B} \cdot d\vec{l} = \oint_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{a}$$

$$\oint_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \mu_0 \oint_S \vec{J} \cdot d\vec{a} \quad \text{-----(3)}$$

$$\text{So we can say: } \vec{\nabla} \times \vec{B} = \mu \vec{J} \quad \text{-----(4)}$$

Taking divergence both side:

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu (\vec{\nabla} \cdot \vec{J}) \quad \text{-----(5)}$$

Since divergence of curl of a vector is always zero i.e. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$

$$\text{therefore: } \mu (\vec{\nabla} \cdot \vec{J}) = 0$$

but from continuity equation

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \text{-----(6)}$$

$$\text{So we can say } \frac{\partial \rho}{\partial t} = 0$$

$$\text{If free current density} = J_f$$

$$\text{And displacement current} = J_D$$

Then

$$J = J_f + J_D \quad \text{-----(7)}$$

$$\text{So from equation (4) i.e } \vec{\nabla} \times \vec{B} = \mu \vec{J}$$

$$\vec{\nabla} \times \vec{B} = \mu (J_f + J_D)$$

Taking divergence both side

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot (\mu (J_f + J_D))$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu (\vec{\nabla} \cdot J_f + \vec{\nabla} \cdot J_D) \quad \text{-----(8)}$$

Since divergence of curl is always zero $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$

$$\vec{\nabla} \cdot J_f + \vec{\nabla} \cdot J_D = 0$$

$$\vec{\nabla} \cdot J_D = -\vec{\nabla} \cdot J_f \quad \text{-----(9)}$$

But from continuity $\vec{\nabla} \cdot J_f + \frac{\partial \rho}{\partial t} = 0$

$$\vec{\nabla} \cdot J_f = -\frac{\partial \rho}{\partial t} \quad \text{-----(10)}$$

Comparing equation (9) and (10)

$$\vec{\nabla} \cdot J_D = \frac{\partial \rho}{\partial t}$$

$$\vec{\nabla} \cdot J_D = \frac{\partial}{\partial t} (\nabla \cdot D)$$

$$J_D = \frac{\partial D}{\partial t}$$

Since $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \oint_S (\vec{J}_f + \vec{J}_D) \cdot d\vec{a}$ from equation (2)

From curl theorem

$$\oint_S (\nabla \times \vec{B}) \cdot d\vec{a} = \mu_0 \oint_S \left(\vec{J}_f + \frac{\partial D}{\partial t} \right) \cdot d\vec{a}$$

So we can write

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \frac{\partial D}{\partial t} \right) \quad \text{since } D = \epsilon E$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}_f + \mu_0 \epsilon \frac{\partial E}{\partial t} \quad \text{This is Maxwell's forth equation.}$$

$$\text{if } \vec{J}_f = 0 \text{ then } \nabla \times \vec{B} = \mu_0 \epsilon \frac{\partial E}{\partial t} \quad \text{for EM waves}$$

POYNTING VECTOR AND POYTING THEOREM:

The rate of flow of wave energy through unit area of space normal to the direction of propagation of EM waves is represented by energy flux vector $\vec{P}$ is also called Poynting Vector.

Energy density for EM waves in a medium is given by:

$$U = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu B^2$$

Therefore total energy $W = \int_V U dV$

And the rate of flow of energy:

$$-\frac{\partial W}{\partial t} = - \int \frac{\partial U}{\partial t} dV$$

$$-\frac{\partial W}{\partial t} = - \int \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu B^2 \right) dV$$

$$-\frac{\partial W}{\partial t} = - \int \left[\frac{1}{2} \epsilon \left(\frac{\partial E^2}{\partial t} \right) + \frac{1}{2} \mu \left(\frac{\partial B^2}{\partial t} \right) \right] dV$$

$$-\frac{\partial W}{\partial t} = - \int \left[\epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu} \vec{B} \frac{\partial \vec{B}}{\partial t} \right] dV \quad \text{-----(1)}$$

Since we know Maxwell's 3rd equation $(\vec{\nabla} \times \vec{E}) = - \frac{\partial \vec{B}}{\partial t}$

Since we know Maxwell's 4th equation $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t}$

Putting value of $\epsilon \frac{\partial \vec{E}}{\partial t}$ and $\frac{\partial \vec{B}}{\partial t}$ in equation (1)

$$-\frac{\partial W}{\partial t} = - \int \left[\frac{1}{\mu} \vec{E} (\vec{\nabla} \times \vec{B}) - \frac{1}{\mu} \vec{B} (\vec{\nabla} \times \vec{E}) \right] dV$$

$$-\frac{\partial W}{\partial t} = \int \frac{1}{\mu} \left[- \vec{E} (\vec{\nabla} \times \vec{B}) + \vec{B} (\vec{\nabla} \times \vec{E}) \right] dV$$

From vector identity i.e $\vec{B} (\vec{\nabla} \times \vec{E}) - \vec{E} (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot [\vec{E} \times \vec{B}]$

$$-\frac{\partial W}{\partial t} = \frac{1}{\mu} \int_V \vec{\nabla} \cdot [\vec{E} \times \vec{B}] dV \quad \text{-----(2)}$$

According to Gauss's divergence theorem

$$\int_V [\vec{\nabla} \cdot \vec{A}] dV = \int_S \vec{A} \cdot d\vec{a}$$

Here In equation (2) $\vec{E} \times \vec{B} = \vec{A}$

Therefore $-\frac{\partial W}{\partial t} = \frac{1}{\mu} \int_S [\vec{E} \times \vec{B}] \cdot d\vec{a} \quad \text{-----(3)}$

Since flow of energy defined as

$$-\frac{\partial w}{\partial t} = \int_S \vec{P} \cdot d\vec{a} \quad \text{-----(4)}$$

From equation 3 (comparing with 4)

$$\vec{P} = \frac{1}{\mu} [\vec{E} \times \vec{B}] \quad \text{Watt/m}^2$$

$$\vec{P} = \frac{1}{\mu} |\vec{E} \times \vec{B}| \hat{n} \quad \text{Where } \hat{n} \text{ is the unit vector normal to the plane of } \vec{E} \text{ and } \vec{B} \text{ and along the direction of propagation of wave.}$$

$$\text{Since } |\vec{B}| = \frac{E}{v}$$

$$\text{Therefore } \vec{P} = \frac{1}{\mu} E B \hat{n} \quad \text{putting value of } |\vec{B}| = \frac{E}{v}$$

$$\vec{P} = \frac{1}{\mu v} E^2 \hat{n} \quad \text{since } v = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\vec{P} = \epsilon v E^2 \hat{n}$$

INTENSITY OF EM WAVES

Intensity is generally defined as the rate of flow of mean wave energy through the unit area normal to the direction of propagation of EM wave

Therefore intensity

$$I = \langle \vec{P} \rangle = \langle \epsilon v E^2 \hat{n} \rangle$$

$$I = \epsilon v \langle E^2 \rangle \hat{n}$$

Since $\vec{E}$ and $\vec{B}$ in EM wave with respect to time is always simple harmonic, therefore the mean value of E^2 is always equals to the $\frac{1}{2} E_0^2$

$$I = \epsilon v \left\langle \frac{1}{2} E_0^2 \right\rangle \hat{n} \quad \text{Watt/m}^2$$