

Electrodynamics

→ The study of moving electric charges and their interaction with magnetic and electric fields.

Fields:- The field on space, in which the physical quantity depends upon the coordinates of the point of observation.

(a) Scalar field ⇒ A scalar quantity depends upon the coordinates of observation.

$$\phi = \phi(x, y, z)$$

(b) Vector field ⇒ If a magnitude or direction or both of vector quantity change with change in position of point of observation, that is vector quantity.

$$\vec{A} = \vec{A}(x, y, z)$$

$$|\vec{A}| = \frac{d\phi}{da} \Rightarrow \boxed{\text{Flux} = d\phi = \vec{A} \cdot d\vec{a}}$$

DEL Operator:- The del operator is the differential operator, which is represented by ∇ & is given by

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

$$\text{Scalar function} = \nabla F = \hat{i} \frac{\partial F}{\partial x} + \hat{j} \frac{\partial F}{\partial y} + \hat{k} \frac{\partial F}{\partial z} \quad (\text{Gradient})$$

→ When it acts on vector function $\vec{A}$ via dot product,

$$\nabla \cdot \vec{A} \quad (\text{divergence of } \vec{A})$$

→ When it acts on vector function $\vec{A}$ via cross product,

$$\nabla \times \vec{A} \quad (\text{curl of } \vec{A})$$

Divergence

The divergence of vector field A is the integral over the closed surface by an infinitesimal volume divided by that volume as the volume shrinks to zero i.e.

$$\text{div } A = \lim_{\Delta V \rightarrow 0} \frac{\oint_S A \cdot d\vec{s}}{\Delta V} \quad \left| \quad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right|$$

Curl

The curl of vector field is the amount of "rotation" or angular momentum of the contents of given region of space.

$$\begin{aligned} \nabla \times \vec{A} &= \text{Curl } A = \lim_{\Delta V \rightarrow 0} \frac{\oint_C A \cdot d\vec{l}}{\Delta V} \cdot \hat{n} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \end{aligned}$$

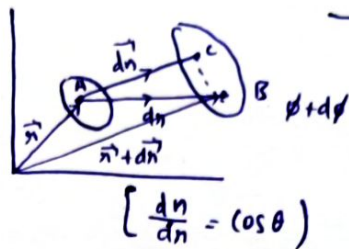
GRADIENT

Del operation

$$\nabla \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right)$$

Divergence:-

The vector flux per unit volume emerging out of closed surface is enclosing a volume element placed in vector field.



$$\text{div}(\vec{A}) = \nabla \cdot \vec{A}$$



$$\nabla \cdot \vec{A} = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z)$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Curl:-

Curl of a vector field is defined as the cross product of ∇ & $\vec{A}$



$$\text{Curl of } \vec{A} = \nabla \times \vec{A}$$

$$\text{curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$



$$\text{Curl } \vec{A} = \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \hat{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\text{Curl } \vec{A} = \lim_{ds \rightarrow 0} \left(\frac{\oint \vec{A} \cdot d\vec{l}}{ds} \right) \hat{n}$$

→ Gradient is the rate of change of a quantity with distance

$$\text{grad}(\phi) = \frac{\partial \phi}{\partial n} \hat{n} \quad \text{--- (1)}$$

$$d\vec{n} = \hat{i} dx + \hat{j} dy + \hat{k} dz \quad \text{--- (2)}$$

$$\text{grad}(\phi) \cdot d\vec{n} = \frac{\partial \phi}{\partial n} \cdot \hat{n} \cdot d\vec{n}$$

$$\therefore \text{as we know } \hat{n} \cdot d\vec{n} = dn \cos \theta = dn$$

$$\text{grad}(\phi) \cdot d\vec{n} = \frac{\partial \phi}{\partial n} \cdot dn = d\phi \quad \text{--- (3)}$$

$$d\phi = \left(\frac{\partial \phi}{\partial x} \right) dx + \left(\frac{\partial \phi}{\partial y} \right) dy + \left(\frac{\partial \phi}{\partial z} \right) dz \quad \text{--- (4)}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z) \cdot (\hat{i} B_x + \hat{j} B_y + \hat{k} B_z) = A_x B_x + A_y B_y + A_z B_z$$

$$d\phi = \left(\hat{i} \frac{d\phi}{dx} + \hat{j} \frac{d\phi}{dy} + \hat{k} \frac{d\phi}{dz} \right) (\hat{i} dx + \hat{j} dy + \hat{k} dz)$$

$$d\phi = \left(\hat{i} \frac{d\phi}{dx} + \hat{j} \frac{d\phi}{dy} + \hat{k} \frac{d\phi}{dz} \right) d\vec{n}$$

$$\text{grad}(\phi) = \nabla \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

LAPLACE and POISSON'S Equation $\Rightarrow$

Poisson equation relates the mass redistribution potential to the displacement field, more specifically to the dilatation & the density gradient.

$$\text{Gauss' Law} \Rightarrow \nabla \cdot D = f_v$$

using relationship $D = eE$

$$\nabla \cdot E = \frac{f_v}{e}$$

We apply the relationship $E = -\nabla V$ (Electric potential & field intensity)

$$\nabla \cdot \nabla V = -\frac{f_v}{e}$$

$$\nabla \cdot \nabla = \left[\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right] \cdot \left[\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right]$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boxed{\nabla^2 V = -\frac{f_v}{e}} \quad \text{Poisson's Eqn}$$

where $V \rightarrow$ Electric potential
 $f_v \rightarrow$ Volume charge density
 $e \rightarrow$ dielectric constant

Laplace :- If an enclosed surface does not have any volume charge f_v , then Poisson's eqn becomes

$$\boxed{\nabla^2 V = 0} \quad \text{Laplace eqn}$$

[$\rightarrow$ Used in boundary value problems.]

Biot-Savart's Law:-

This law deals with magnetic field at a point due to small current element.

According to law magnitude of magnetic field dB at point P due to current element depends upon

$$dB \propto I, \quad dB \propto dI, \quad dB \propto \sin \theta, \quad dB \propto \frac{1}{r^2}$$

$$dB \propto \frac{Idl \sin \theta}{r^2}$$

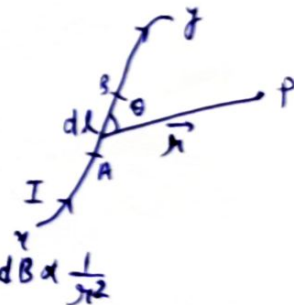
$$\boxed{dB = k \frac{Idl \sin \theta}{r^2}}$$

Where $k =$ proportionality constant

$$k = \frac{\mu_0}{4\pi}$$

$\mu_0 =$ magnetic permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb} \cdot \text{A}^{-1} \text{m}^{-1} \text{ or Tesla A}^{-1} \text{m}.$$



Magnetic field in Vector form $\Rightarrow$

$$d\vec{B} = k I \frac{d\vec{l} \times \vec{r}}{r^3} \quad \text{on } \vec{B} = \frac{1}{4\pi} \int \frac{Id\vec{l} \times \mu_0}{(R^2 + r^2)} \cdot \frac{R}{(R^2 + r^2)^{3/2}}$$

$$B = \frac{\mu_0 i R^2}{2(R^2 + r^2)^{3/2}}$$

Divergence of Magnetic field $\Rightarrow$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V (\vec{J} \times \frac{\vec{r}}{r^3}) dV$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int_V \vec{\nabla} \cdot (\vec{J} \times \frac{\vec{r}}{r^3}) dV$$

Now using vector triple product

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot (\vec{a} \times \vec{b}) - \vec{b} \cdot (\vec{a} \times \vec{c})$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \oint_V \left[\frac{\vec{r}}{r^3} (\vec{\nabla} \times \vec{J}) - \vec{J} (\vec{\nabla} \times \frac{\vec{r}}{r^3}) \right] dV$$

$$\vec{\nabla} \times \vec{J} = 0$$

$$\vec{\nabla} \cdot \vec{B} = -\frac{\mu_0}{4\pi} \oint_V [\vec{J} \cdot (\vec{\nabla} \times \frac{\vec{r}}{r^3})] dV$$

$$\vec{\nabla} \cdot \vec{B} = -\frac{\mu_0}{4\pi} \oint_V [\vec{J} \cdot (-3\vec{r} r^{-5} \vec{r}) \times \vec{r} + r^{-3} (0)] dV$$

$$\vec{\nabla} \cdot \vec{B} = -\frac{\mu_0}{4\pi} \oint_V [\vec{J} \cdot 0] dV$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

Curl of Magnetic field:-

from Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint (\text{curl } \vec{B}) \cdot d\vec{s} = \oint \vec{B} \cdot d\vec{l} \quad \& \quad J = \frac{I}{S} \quad \text{on } I = \oint \vec{J} \cdot d\vec{s}$$

$$\oint (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{s}$$

$$\oint \vec{\nabla} \times \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{s}$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

Faraday's Law $\Rightarrow$ According to Faraday's law of induction

that an EMF is induced in the closed loop when number of magnetic field lines changes.

$$e = -\frac{d\phi}{dt} \quad \text{--- (1)}$$

$$\phi = \oint_S \vec{B} \cdot d\vec{s} \quad \text{also} \quad e = \oint_L \vec{E} \cdot d\vec{l}$$

$$\oint_L \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_S \vec{B} \cdot d\vec{s}$$

$$\oint_S (\text{curl } \vec{E}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s}$$

(Applying Stokes theorem)

$$\oint_L \vec{A} \cdot d\vec{l} = \oint_S (\text{curl } \vec{A}) \cdot d\vec{s}$$

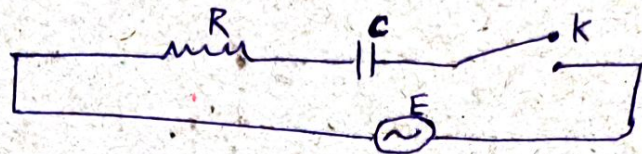
So we can say

$$\boxed{\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Faraday's law for EMF.

Displacement Current $\Rightarrow$ As per experiment conducted by Maxwell the magnetic fields are generated not only due to flow of current but also due to time dependent changes in the electric field.

So this type of time varying field is equivalent to current called "Displacement current".



$$\vec{E} = \frac{\sigma}{\epsilon_0} \quad (\sigma = \text{charge density})$$

$$\underline{D = \epsilon_0 \vec{E} = \sigma} \quad \text{--- (1)} \quad (\text{Displacement vector f. field})$$

$$I = \frac{\partial q}{\partial t}$$

$$\frac{I}{A} = \frac{\partial}{\partial t} \left(\frac{q}{A} \right) \quad \Rightarrow \quad J = \frac{I}{A}, \quad \sigma = \frac{q}{S}$$

$$\vec{J} = \frac{\partial}{\partial t} (\Rightarrow)$$

from eqⁿ - (1)

$$\vec{J} = \frac{\partial}{\partial t} (\vec{D}) = \vec{J}_d$$

So we can say rate of change of displacement vector is equivalent to current density. It is called Displacement current Density.

Maxwell's Equations

Maxwell's equations are a set of 4 partial differential equations that relate the electric & magnetic fields to their sources, charge density & current density. These equations can be combined to show that light is an electromagnetic wave.

The Maxwell's equations are:

1. Gauss's Law for Electric Fields
2. Gauss's Law for Magnetic Fields
3. Faraday's Law of Magnetic Induction
4. Ampere's Law plus Maxwell's Displacement Current

1. Maxwell First Equation (Gauss's Law for Electric Fields) $\Rightarrow$ According to this law, total electric flux passing through a closed surface in vacuum is equal to the product of total charge (Q) contained inside & $1/\epsilon_0$.

$$\text{i.e. } \oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0} = \frac{Q}{\epsilon} \quad \text{--- (1)}$$

If charge inside the closed surface is continuously distributed & ρ is charge volume density, then we can write

$$Q = \int_V \rho dv \quad \text{--- (2)}$$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon} \int_V \rho dv \quad \text{--- (3) (integral form of Maxwell's I eqn)}$$

Using Gauss's divergence theorem

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E}) \cdot dv$$

$$\int_V (\vec{\nabla} \cdot \vec{E}) dv = \frac{1}{\epsilon} \int_V \rho dv \Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}} \quad \text{on} \quad \boxed{\text{div}(\vec{E}) = \frac{\rho}{\epsilon}} \quad (\text{Differential form})$$

If medium is homogeneous & isotropic, then

$$\nabla \cdot (\epsilon \vec{E}) = \rho \Rightarrow \boxed{\nabla \cdot \vec{D} = \rho} \quad \text{--- (5)} \quad \text{where } \vec{D} = \epsilon \vec{E} \text{ (displacement vector)}$$

For free space $\rho = 0$

$$\oint_S \vec{E} \cdot d\vec{a} = 0 \quad \text{--- (7)} \Rightarrow \nabla \cdot \vec{E} = 0 \quad \text{--- (8)} \quad \text{on } \boxed{\nabla \cdot \vec{E} = 0}$$

2. Maxwell's Second Equation $\Rightarrow$
(Gauss's law for magnetic field)

According to law, net outgoing magnetic flux from any surface is always zero. This is due to fact that magnetic field lines always form closed loop.

i.e. $\phi_B = 0 \Rightarrow \boxed{\oint_S \vec{B} \cdot d\vec{a} = 0}$ (Integral form of Maxwell's II Eqn)

Using Gauss's divergence theorem

$$\oint_S \vec{B} \cdot d\vec{a} = \int_V (\text{div } \vec{B}) dV \Rightarrow \boxed{\text{div } \vec{B} = 0} \quad \text{on } \boxed{\nabla \cdot \vec{B} = 0} \quad \text{--- (Differential form of Maxwell's II)}$$

(This equation prove theoretically that magnetic monopole does not exist in nature)

3. Maxwell's Third Equation $\Rightarrow$
(Faraday's Law of Magnetic Induction)

According to the law, whenever magnetic flux linked with a circuit changes, an e.m.f is induced in the circuit & this induced e.m.f is directly proportional to the rate of change of magnetic flux linked with the circuit.

According to Lenz's Law, it's direction is always opposite to the change responsible for it's production.

i.e.
$$e = - \frac{\partial \phi_B}{\partial t}$$

Since, line integral of electric field is also equal to the e.m.f.

$$e = \oint_c \vec{E} \cdot d\vec{n}$$

We know that magnetic flux

$$\oint_c \vec{E} \cdot d\vec{n} = -\frac{\partial}{\partial t} \oint_s \vec{B} \cdot d\vec{a}$$

$$\phi_B = \oint_s \vec{B} \cdot d\vec{a}$$

$$\text{on } \boxed{\oint_c \vec{E} \cdot d\vec{n} = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}} \quad (\text{Integral form of Maxwell's III equation})$$

Using Stoke's curl theorem

$$\oint_c \vec{E} \cdot d\vec{n} = -\oint_s (\nabla \times \vec{E}) \cdot d\vec{a}$$

$$\text{So } \oint_s (\nabla \times \vec{E}) \cdot d\vec{a} = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{on } \boxed{\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad (\text{Differential form of Maxwell's III Equation})$$

For static field (w.r.t. time)

$$\frac{\partial \vec{B}}{\partial t} = 0 = \frac{\partial \vec{E}}{\partial t} \Rightarrow \nabla \times \vec{E} = 0 \quad \text{on } \boxed{\oint_s \vec{E} \cdot d\vec{n} = 0}$$

4. Maxwell's IV Equation $\Rightarrow$
(Ampere's circuit law)

According to the law, line integral of magnetic field over a closed path is equal to μ times the net current passes through the closed path.

$$\oint_c \vec{B} \cdot d\vec{n} = \mu \sum I$$

$$\text{If current density is } \vec{J} \text{ then } \sum I = \oint_s \vec{J} \cdot d\vec{a} \quad \text{so } \oint_c \vec{B} \cdot d\vec{n} = \mu \oint_s \vec{J} \cdot d\vec{a} \quad \text{--- (1)}$$

from Stoke's theorem

$$\oint_c \vec{B} \cdot d\vec{n} = \oint_s (\nabla \times \vec{B}) \cdot d\vec{a} \quad \text{--- (2)}$$

$$\nabla \times \vec{B} = \mu \vec{J} \quad \text{--- (3)}$$

$$\nabla \cdot (\nabla \times \vec{B}) = \mu (\nabla \cdot \vec{J}) \quad [\text{inconsistency}]$$

But, we know that divergence of curl of vector is always zero.

$$\nabla (\nabla \cdot \vec{B}) = 0$$

$$\nabla \cdot \vec{J} = 0 \quad \text{--- (4) But, from continuity eqn}$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \text{--- (5)}$$

from eqn - (4) & (5) $\frac{\partial \rho}{\partial t} = 0$

Maxwell removed this inconsistency by giving suggestion that total current density $\vec{J}$ can be considered as made of two type of currents; free current density $\vec{J}_f$ & displacement current density $\vec{J}_d$

$$\vec{J} = \vec{J}_f + \vec{J}_d \Rightarrow \nabla \times \vec{B} = \mu (\vec{J}_f + \vec{J}_d) \quad \text{eqn (7.100) now, becomes}$$

$$\nabla \cdot (\nabla \times \vec{B}) = \mu (\nabla \cdot \vec{J}_f + \nabla \cdot \vec{J}_d) \Rightarrow \nabla \cdot \vec{J}_f + \nabla \cdot \vec{J}_d = 0 \Rightarrow \nabla \cdot \vec{J}_f = -\nabla \cdot \vec{J}_d \quad \text{--- (6)}$$

But from continuity eqn

$$\nabla \cdot \vec{J}_f + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla \cdot \vec{J}_f = -\frac{\partial \rho}{\partial t} \quad \text{--- (7)} \Rightarrow \nabla \cdot \vec{J}_d = \frac{\partial \rho}{\partial t}$$

from Maxwell 1st eqn

$$\nabla \cdot \vec{J}_d = \frac{\partial \rho}{\partial t} (\nabla \cdot \vec{B}) \Rightarrow \vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad \text{--- (8)}$$

After Maxwell's modification

$$\oint_C \vec{B} \cdot d\vec{r} = \mu \oint_S \vec{J}_f \cdot d\vec{a} + \vec{J}_d \cdot d\vec{a} \Rightarrow \oint_C \vec{B} \cdot d\vec{r} = \mu \oint_S \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{a} \quad (\text{integral form})$$

Using Stoke's curl theorem

$$\oint_C \vec{B} \cdot d\vec{r} = \oint_S (\nabla \times \vec{B}) \cdot d\vec{a}$$

$$\oint_S (\nabla \times \vec{B}) \cdot d\vec{a} = \mu \oint_S \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{a} \Rightarrow \nabla \times \vec{B} = \mu \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right)$$

$$\text{on } \boxed{\nabla \times \vec{B} = \mu \vec{J}_f + \mu \epsilon \frac{\partial \vec{E}}{\partial t}} \quad \star$$

(differential form)

Maxwell 4th eqn

Poynting Vector & Poynting Theorem $\Rightarrow$

The rate of flow of wave energy through unit area of space normal to direction of propagation of EM waves is represented by energy flux vector $\vec{P}$ is also called Poynting Vector.

Energy density for EM waves in medium: —

$$U = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu B^2$$

$$W = \int_V U dv \quad (\text{total energy})$$

$$-\frac{\partial W}{\partial t} = - \int \frac{\partial U}{\partial t} dv \quad (\text{rate of flow of energy})$$

$$-\frac{\partial W}{\partial t} = - \int \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu B^2 \right) dv$$

$$-\frac{\partial W}{\partial t} = - \int \left[\epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu} \vec{B} \frac{\partial \vec{B}}{\partial t} \right] dv \quad \text{--- (1)}$$

from Maxwell 3rd eqⁿ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

from Maxwell 4th eqⁿ $\nabla \times \vec{B} = \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t}$

$$-\frac{\partial W}{\partial t} = - \int \left[\frac{1}{\mu} \vec{E} (\nabla \times \vec{B}) - \frac{1}{\mu} \vec{B} (\nabla \times \vec{E}) \right] dv$$

$$-\frac{\partial W}{\partial t} = - \int \frac{1}{\mu} \left[-\vec{E} (\nabla \times \vec{B}) - \vec{B} (\nabla \times \vec{E}) \right] dv$$

from vector identity $\vec{B} (\nabla \times \vec{E}) - \vec{E} (\nabla \times \vec{B}) = \nabla \cdot [\vec{E} \times \vec{B}]$

$$-\frac{\partial W}{\partial t} = \frac{1}{\mu} \int_V \nabla \cdot [\vec{E} \times \vec{B}] dv \quad \text{--- (2)}$$

According to Gauss's divergence

$$\int_V [\nabla \cdot \vec{A}] dv = \int_S \vec{A} \cdot d\vec{a}$$

Here in eqⁿ (1) $\vec{E} \times \vec{B} = \vec{A}$

$$-\frac{\partial W}{\partial t} = \frac{1}{\mu} \int_S [\vec{E} \times \vec{B}] \cdot d\vec{a} \quad \text{--- (3)}$$

$$-\frac{\partial W}{\partial t} = \int_S \vec{P} \cdot d\vec{a} \quad \text{--- (4)} \quad \left(\vec{P} = \frac{1}{\mu} [\vec{E} \times \vec{B}] \right)$$

since $|\vec{B}| = \frac{E}{v}$, $\vec{P} = \frac{1}{\mu} E B \hat{n}$ & $|\vec{B}| = \frac{E}{v}$

$$\vec{P} = \frac{1}{\mu v} E^2 \hat{n}$$

since $v = \frac{1}{\sqrt{\mu \epsilon}}$

$$\vec{P} = \epsilon v E^2 \hat{n}$$

Intensity of EM waves: —

rate of flow of mean wave energy through unit area normal to direction of propagation of EM wave.

$$I = \langle \vec{P} \rangle = \langle \epsilon v E^2 \hat{n} \rangle$$

$$I = \epsilon v \langle E^2 \rangle \hat{n}$$

since $\vec{E}$ & $\vec{B}$ is always harmonic so mean value of E^2 is always equal to $\frac{1}{2} E_0^2$ (w.r.t)

$$I = \epsilon v \frac{1}{2} E_0^2 \hat{n} \quad \text{watt/m}^2$$