

resolved if the central maxima of diffraction pattern of one coincide with the 1st secondary minima of other and vice-versa.

$$\text{Resolving power} \propto \frac{1}{\text{Resolution limit}}$$

The Resolving Power of A Diffraction Grating is

$$R = \frac{\lambda}{d\lambda} = nN = \frac{nW}{(a+b)}$$

n = order of spectrum; $(a+b)$ = grating element,
 N = number of ruled lines on grating surface,
 W = width of ruled surface of grating,
 λ = mean value of two spectral lines,
 $d\lambda$ = wavelength difference of two spectral line,
 minimum difference at which lines are seen just at separate.

Bragg's Law

$$2d \sin \phi = n\lambda$$

d = interplanar spacing,

λ = wavelength of x-rays

n = order of spectrum of x-rays

ϕ = glancing angle of incident x-ray.

YEARS QUESTIONS

We know that the direction of principal maxima in plane diffraction grating spectrum is given by

$$(e+d)\sin \theta = n\lambda \quad \dots (1)$$

where n is order of the maximum,

Also, the direction of minima in a single slit pattern is given by

$$e \sin \theta = m\lambda \quad m = 1, 2, 3, \dots \quad \dots (2)$$

If both the conditions (equation 1 and 2) are simultaneously satisfied, particular maximum of order n will be missing or absent in the grating spectrum. These are known as missing order spectrum or absent spectra.

Dividing eq. (1) by eq. (2) we get

$$\frac{e+d}{e} = \frac{n}{m} \quad \dots (3)$$

The above equation (3) is the condition for the spectrum of the order n to be missing or absent.

Prob.3 State Law of Malus.

[R.T.U. 2018]

Sol. Malus's law : It states that the intensity of polarised light transmitted through the analyser varies as a square of cosine of the angle between the plane of transmission of analyser and polariser.

$$I \propto \cos^2 \theta; I = I_0 \cos^2 \theta.$$

Prob.4 On what factors does the dispersive power of a grating depend? [R.T.U. 2017]

Sol. Dispersive power of a grating

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\left(\frac{a+b}{n}\right)^2 - \lambda^2}}$$

Hence dispersive power of a grating depends on grating element $(a+b)$, order of spectrum (n) and the wavelength of light (λ) .

Prob.5 What is the optic axis and the principle section of a crystal? [R.T.U. 2017]

Sol. The calcite and quartz crystals, has an important characteristic i.e. there is only one direction through the crystal along which no double refraction takes place. This particular direction is called **optic axis**.

A plane containing the optic axis and $\perp$ to the face of the crystal is called the **principal plane** or **principal section** for that face.

Prob.6 A glass lens $\mu = 1.45$ is coated with MgF_2 film having dielectric constant 1.38 to increase the transmission of normally incident yellow light $\lambda = 5800 \text{ \AA}$. Calculate the minimum thickness of the film that should be deposited on the lens.

Sol. Minimum thickness of film is given by

$$t_{\min} = \frac{\lambda}{4\mu_f}$$

$$\text{Given: } \lambda = 5800 \text{ \AA} = 5800 \times 10^{-10} \text{ m}$$

$$\mu_f = 1.38$$

$$t_{\min} = \frac{5800 \times 10^{-10}}{4 \times 1.38} = 1050.7 \text{ \AA}.$$

Prob.7 What is light?

Sol. Light is an electromagnetic radiation refers to the visible region of the electromagnetic spectrum corresponding to wavelength range from $4000 \text{ \AA}$ - $8000 \text{ \AA}$. A charged particle can radiate electromagnetic radiation only if it is undergoing accelerated motion.

Prob.8 State the principle of superposition.

Sol. Whenever two or more waves superimpose (meet), the instantaneous total displacement at any point is equal to the vector sum of the individual displacements of wave at that point.

Prob.9 What is the function of compensating plate in michelson interferometer?

Sol. The reflected beam moving towards M_1 crosses the beam splitter plate P_1 (in medium) twice before interfering, while the transmitted beam doesn't pass even once in the absence of plate P_2 that lies totally in air. Hence due to this, an extra optical path $2(\mu - 1)t$ (air) is introduced in the reflected beam. (Here t is the thickness of the plate and μ is the refractive index of the material). This path difference is compensated by using a compensating plate P_2 in the path of the transmitted beam.

Prob.10 In michelson interferometer 200 fringes cross the field of view when the movable mirror is displaced through 0.05896 mm . Calculate the wavelength of the monochromatic light used.

Sol. In Michelson Interferometer

$$\lambda = \frac{2x}{N}$$

$$\text{Given } x = 0.05893 \text{ mm} = 0.05893 \times 10^{-3} \text{ m}$$

$$\text{and } N = 200$$

$$\therefore \lambda = \frac{2 \times 0.05896 \times 10^{-3}}{200}$$

$$\Rightarrow \lambda = 5896 \times 10^{-10} \text{ m} = 5896 \text{ \AA}$$

Prob. 11 Define the dispersive power of a grating.

Sol. Dispersive power of a grating is defined as the rate of change of angle of diffraction corresponding to the wavelength of light.

If the wavelength changes from λ to $\lambda + d\lambda$ and the corresponding change in angle of diffraction is from θ to $\theta + d\theta$ then the rate $\frac{d\theta}{d\lambda}$ is known as dispersive power of grating.

Prob. 12 How do you understand by resolving power of a microscope?

Sol. The resolving power of a microscope is defined as the reciprocal of the smallest separation at which two objects can be seen just as separate. The resolving power of a microscope is a function of the wavelength of light used and the numerical aperture (NA), which is characteristic of the lens system and the medium between the lens and the specimen.

II. LONG ANSWER TYPE QUESTIONS

Prob. 13 (a) Draw labeled diagram of a Michelson's Interferometer. How shall we use it to measure wavelength of a monochromatic source of light? Explain the role of compensating glass plate in Michelson's Interferometer.

(b) In Michelson's Interferometer the reading of a pair of maximum distinctness were found to be 0.6989 mm and 0.9334 mm. If mean wavelength of the doublet used is 5893 Å, find the difference between wavelengths of components.

[R.T.U. 2018]

Sol. (a) Working of a Michelson's Interferometer : Michelson designed an instrument for the measurement of wavelength of sodium light, thickness of thin film and for many applications. This instrument is based on principle of interference of light known as Michelson's Interferometer.

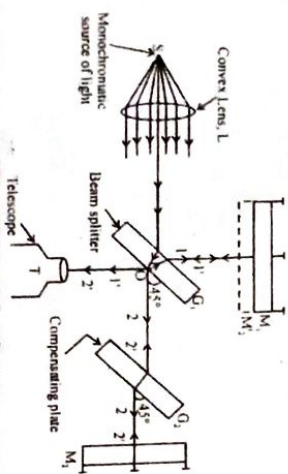


Fig. 1 : Michelson's Interferometer

It is based on principle of interference of light by the way of division of amplitude. According to this the incident beam is divided into two parts and sent into two perpendicular directions and brought back together by using plane mirror to interfere each other. A schematic diagram of Michelson's Interferometer is shown in Fig. 1.

Construction : It consists of two highly polished plane mirror M_1 and M_2 with two optically plane glass plate G_1 and G_2 which are of same material and same thickness. The mirror M_1 and M_2 are adjusted in such a way that they are mutually perpendicular to each other. The plate G_1 and G_2 are exactly parallel to each other and placed at 45° to mirror M_1 and M_2 . Plate G_1 is half silvered from its back while G_2 is plane and act as compensating plate. Plate G_1 is known as beam-splitter plate.

The mirror M_1 with screw on its back can slightly tilted about vertical and horizontal direction to make it exactly perpendicular to mirror M_2 . The mirror M_1 can be moved forward or backward with the help of micrometer screw and this movement can be measured very accurately.

Working : Light from a broad source is made parallel by using a convex lens L. Light from lens L is made to fall on glass plate G_1 which is half silver polished from its back. This plate divide the incident beam into two light rays by the partial reflection and partial transmission, known as Beam splitter plate. The reflected ray travels towards mirror M_1 and transmitted ray towards mirror M_2 . These rays after reflection from their respective mirrors meet again at 'O' and superpose to each other to produce interference fringes. This fringe pattern is observed by using telescope.

Functioning of Compensating Plate : In absence of plate G_2 , the reflected ray passes the plate G_1 twice, whereas the transmitted ray does not pass even once. Therefore, the optical path of the two rays are not equal. To equalize this path the plate G_2 which is exactly same as the plate G_1 is introduced in path of the ray proceeding towards mirror M_2 , that is why this plate is called compensating plate because it compensate the additional path difference.

Formation of Circular Fringes : The shape of fringes in M.I. depends on inclination of mirror M_1 and M_2 .

Circular fringes are produced with monochromatic light, if the mirror M_1 and M_2 are perfectly perpendicular to each other. In this position an image of mirror M_2 , M_2' is formed due to half silvered polished plate G_1 , just below the mirror M_1 . The virtual image of mirror M_2 and the mirror M_1 must be parallel. Therefore, it is assumed that an imaginary air film is formed in between mirror M_1 and virtual image mirror M_2' .

The interference pattern can be considered as the rays of light reflected, from the surface of mirror M_1 (real) and mirror M_2' (virtual). Therefore, the interference pattern will be obtained due to imaginary air film enclosed between M_1 and M_2' .

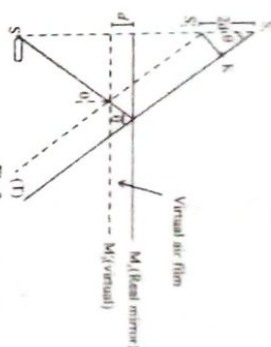


Fig. 2

From Fig. 2 if the distance M_1 and M_2' is 'd', the distance between S_1 and S_2 will be $2d$.

Where, S_2 = Image of S due to M_2'

S_1 = Image of S due to M_1

The path difference between the two beam will be

$$\Delta = 2d \cos \theta \left\{ \frac{S_1 K}{S_1' S_2'} = \cos \theta \right\}, [S_1' S_2' = 2d]$$

Therefore, the rays for which path difference

$$2d \cos \theta = n\lambda \text{ will produce maxima.}$$

These circular fringes which are due to interference with a phase difference determined by the inclination ' θ ' are known as fringes of equal inclination.

Circular fringes can be seen by telescope because they are formed at infinity because they are formed due to two parallel interfering rays. When θ becomes zero, the whole pattern becomes dark. Since a circular fringe is formed at the same inclination so they are called fringe of equal inclination and also called Haidinger's fringes.

Determination of the Difference in Wavelength of D_1 and D_2 Lines of Sodium Light by Michelson Interferometer

For the measurement of difference in wavelength of two closely spaced sodium lines D_1 and D_2 , firstly the M_1 is adjusted for circular fringes. For this purpose mirror M_1 and M_2 are made exactly perpendicular to each other and are equidistant from the beam splitter P. As we know that there are two spectral lines of sodium light D_1 and D_2 and they are very close to each other and the difference in their wavelength is the order of 6Å.

Suppose the wavelength of D_1 line is λ_1 and the wavelength of D_2 line is λ_2 . Each spectral line will give rise to its own fringe pattern in Michelson Interferometer. By adjusting the position of the mirror M_1 of the Michelson Interferometer, a position is found where the fringes are very bright. In this position, the bright fringe due to D_1 line coincides with the bright fringe due to D_2 line. When the mirror M_1 is moved, the two sets of fringes get out of step because their wavelengths are different.

When the mirror M_1 has been moved through a certain distance, the bright fringe due to wavelength λ_1 coincide with dark fringe due to wavelength λ_2 and no fringes will be seen in this case. On further moving mirror M_1 , a position is found when a bright fringe of wavelength λ_1 falls on the bright fringe of the wavelength λ_2 and the fringes are again distinct. This is only possible when the $(n+1)^{\text{th}}$ order of the longer wavelength coincides with the n^{th} order of the shorter wavelength.

Let n_1 and n_2 be the changes in the order at the centre of the fields when the mirror M_1 is moved through a distance d between two successive positions of maximum distinctness of the fringes.

$$2d = n_1 \lambda_1 = n_2 \lambda_2 \quad \dots (1)$$

If λ_1 is greater than λ_2 ($\lambda_1 > \lambda_2$)

$$n_2 = n_1 + 1$$

Therefore

$$2d = n_1 \lambda_1 = (n_1 + 1) \lambda_2 \quad \dots (2)$$

$$n_1 \lambda_1 = (n_1 + 1) \lambda_2$$

$$n_1 (\lambda_1 - \lambda_2) = \lambda_2$$

$$n_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$\dots (3)$$

Putting the value of n_1 in equation (1), we get

$$2d = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2d} \quad \therefore \lambda_2 = \lambda_1 \lambda_2$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_2^2}{2d} \quad \dots (4)$$

Taking λ_{av} as the mean of λ_1 and λ_2

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{1}{2d} \quad \dots (5)$$

Hence the difference in wavelength ($\lambda_1 - \lambda_2$) can be calculated in terms of wave number,

$$\text{Since } \nu_1 = \frac{1}{\lambda_1}, \nu_2 = \frac{1}{\lambda_2}$$

$$\nu_2 - \nu_1 = \frac{1}{2d} \quad \dots (6)$$

This equation represents the difference in the wave numbers of the two spectral lines.

Wavelength of a Monochromatic Source of Light:

Monochromatic light is allowed to fall on plate G_1 . The

M₁ is adjusted for circular fringes. For this mirror M₁ is made exactly perpendicular to mirror M₂, with the help of levelling screw and movable mirror M₁ is moved in such a way so that $G_1 M_2 = G_1 M_1$.

Consequently circular fringes are observed when viewed through telescope.

Suppose the separation between real mirror M₁ and image of mirror M₂, M_{1'} is such that an n^{th} order dark ring is formed at the centre in the field of view. Thus the conditions for dark fringe at centre

$$2d \cos \theta = n\lambda, \text{ here } \theta = 0^\circ$$

$$2d = n\lambda$$

adding λ on both sides

$$2d + \lambda = n\lambda + \lambda$$

$$2 \left(d + \frac{\lambda}{2} \right) = (n+1)\lambda$$

$$\dots (2)$$

From equation (2) it is seen that when value of $d \rightarrow d + \frac{\lambda}{2}$, then $n \rightarrow (n+1)$.

In other words, when mirror M₁ is moved by a distance $\frac{\lambda}{2}$ one fringe is shifted in view of telescope T.

Now mirror M₁ is moved and number of fringes shifted in telescope is counted.

Let on moving mirror M₁ through x distance the number of fringes shifted is n .

On moving mirror by $\frac{\lambda}{2}$ number of fringe shift = 1.

On moving mirror by x distance number of fringe shift

$$n = \frac{1}{\lambda/2} \cdot x : n = \frac{2x}{\lambda} \Rightarrow \lambda = \frac{2x}{n}$$

If we know n and x , we can find λ .

Sol.(b) In Michelson's interferometer

$$(\lambda_1 - \lambda_2) = \frac{\lambda^2}{2x}$$

$$\text{Given: } \lambda = 5893 \text{ \AA} = 5.893 \times 10^{-7} \text{ m}$$

$$x = 0.9334 - 0.6989 = 0.2345 \text{ mm} = 0.2345 \times 10^{-3} \text{ m}$$

$$(\lambda_1 - \lambda_2) = \frac{(5.893 \times 10^{-7})^2}{2 \times 0.2345 \times 10^{-3}}$$

$$= 7.4108 \times 10^{-10} \text{ m}$$

$$= 7.4108 \text{ \AA}$$

Prob. 2. Explain Rayleigh's criterion of resolution. What is meant by resolving power of a grating? Derive an expression for resolving power of grating & on what factor does it depend.

[R.T.U. 2018]

OR

Explain Rayleigh's criterion of just resolution of two spectral lines of equal intensities giving suitable intensity distribution curves. Show how the resolving power of a plane transmission grating depends on:

- (i) Number of ruled lines
(ii) Width of ruled space

[R.T.U. Jan. 2016]

OR

Explain Rayleigh's criterion of just resolution of two spectral lines of equal intensities giving suitable intensity distribution curves. Show how the resolving power of a plane transmission grating depends on:

- (i) Number of ruled lines
(ii) Width of ruled surfaces (space)

Deduce suitable formula in support of your answer.

[R.T.U. 2010, Raj. Univ. 2005]

OR

What do you understand by 'resolution'? Explain. What is meant by resolving power of a grating? Deduce an expression for the same, and discuss its dependence on various constant of grating.

[Raj. Univ. 2002]

OR

Explain Rayleigh criterion for resolution and apply it to distinguish between the resolving power and dispersive power of a diffraction grating.

[R.T.U. 2009]

Sol. Resolving Power: The resolving power of an optical instrument (like human eye or a telescope) is its ability to separate far-away objects that are close together into individual images, as opposed to a single merged image. If we look at two stars in the sky, separated by a large enough angle one can say very easily that they are two stars by looking at them through a telescope. The telescope resolves these two stars into separate objects while our eye cannot resolve them and they look like one star to us, this is all because of diffraction. Similarly if we look at the headlights of a car as it backs away from us to a far distance when the car is close, it is easy to distinguish two separate headlights. But as it gets further away, it's harder to resolve the two headlights. Finally, there is a certain point when the car gets even further away, that we can not distinguish the two headlights clearly. This inability to resolve two closely spaced objects is due to diffraction. Thus to see two close objects just as separate is the resolution. In microscopes and telescopes the geometrical positions of two near by point objects is resolved, which is known as geometrical resolution. Whereas in case of diffraction grating we resolve two near by spectral lines which are very close to each other this is known as spectral resolution.

The resolving power of an instrument is the ability of an optical instrument to distinguish (or resolve) two closely spaced objects.

(a) Resolving Power of the Telescope: The resolving power of a telescope is defined as the minimum angle between two objects of equal magnitude (intensity) to see them just as separate, i.e. the 'smallest angular separation' that can be unambiguously distinguished is called the resolving power of the telescope and is proportional to the ratio of the wavelength of light being observed to the diameter of the telescope. Thus, the larger the diameter, the smaller the minimum angular separation and the higher the resolving power.

(b) Resolving Power of a Microscope: The resolving power of a microscope is defined as the smallest separation at which two separate objects can be distinguished (or resolved). The resolving power of a microscope is a function of the wavelength of light used and the Numerical Aperture (NA), which is characteristic of the lens system and the medium between the lens and the specimen.

Rayleigh Criterion of Resolution: In order to express the resolving power of an optical instrument Lord Rayleigh proposed a universal criterion known as Rayleigh's criterion of resolution. This criterion is the generally accepted criterion for the minimum resolvable detail. According to Rayleigh two nearby point source images are said to be just resolved if the position of the central maxima of one point source coincides (overlaps) with the minimum of another point source. It is possible to resolve the two objects as long as the central maxima of the two diffraction patterns do not overlap. If the two central maxima overlap the two objects look like one.

Let P and Q be the principal (central) maxima of diffraction pattern of two spectral lines of wavelength say λ and $\lambda + d\lambda$. If the difference in the angle of diffraction is quite large then two maxima can be seen distinctly and both the spectral lines will appear well resolved. As shown in fig. (a).

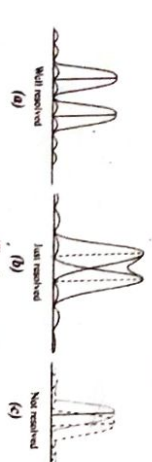


Fig. 1

Now if the difference in wavelength of spectral lines decreases both the spectral lines get closer. A situation comes when the positions of principal maxima of spectral line of wavelength λ coincides (overlap) with the position of spectral line of wavelength $\lambda + d\lambda$, or vice-versa. Then the resultant intensity of both diffraction patterns will only be observed. This is known as condition of just-resolution. The resultant

intensity curve is shown in fig. (b). A dip in the intensity curve in the middle of central maxima of P and central maxima of Q is found, where the intensity is approximately 20% less than that of any intensity peak individually. This position of spectral lines where the intensity in the middle is 80% approximately of any spectral lines is known as condition of just resolution for spectral lines, also known as spectral resolution.

Now if the difference in wavelength $d\lambda$ is decreases then the diffraction pattern of both the spectral lines collapse to each other and resultant intensity curves show that there is only one maxima. Hence in result we are not able to resolve these spectral lines. As shown in fig. (c).

Formula for the Resolving Power of Grating:

The resolving power of a grating is defined as its ability to show two spectral lines of very close wavelengths as separate.

It is measured in terms of the ratio $\lambda/d\lambda$. λ is wavelength of a spectral line and $d\lambda$ is the difference in wavelength between this line and a neighboring line which is just resolved.

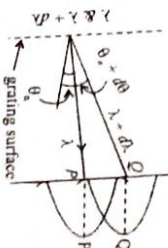


Fig. 2

Consider a beam of light consisting of wavelengths λ and $\lambda + d\lambda$, is incident normally on a grating surface.

Let θ_n and $\theta_n + d\theta_n$ is the direction of principal maxima of wavelength λ and $\lambda + d\lambda$, then

$$(a+b) \sin \theta_n = n\lambda \quad \dots (1)$$

$$(a+b) \sin (\theta_n + d\theta_n) = n(\lambda + d\lambda) \quad \dots (2)$$

Let the n^{th} principal maxima of wavelength λ is formed at P, if 1st minima of this wavelength is formed in the direction $(\theta_n + d\theta_n)$ i.e. at point Q, then according to Rayleigh's criterion the spectral line of wavelength λ and $\lambda + d\lambda$ will resolve.

Condition of 1st minima of wavelength λ in direction $(\theta_n + d\theta_n)$ i.e. at Q,

$$N(a+b) \sin (\theta_n + d\theta_n) = m\lambda \quad \dots (3)$$

$$\text{For minimum } m = N\lambda + 1 \quad \dots (4)$$

$$N = \text{Total number of ruled lines on grating}$$

$$N(a+b) \sin (\theta_n + d\theta_n) = (N\lambda + 1)\lambda$$

$$(a + b) \sin \theta_n + d \sin \theta = \frac{(n\lambda + 1)}{N} \lambda = n\lambda + \frac{\lambda}{N} \quad \dots (5)$$

Using (2) and (5), we have

$$n\lambda + \frac{\lambda}{N} = n\lambda + d \sin \theta + \frac{\lambda}{N} \quad \dots (6)$$

$$n\lambda + \frac{\lambda}{N} = n\lambda + d \sin \theta + \frac{\lambda}{N}$$

Resolving Power = Order of spectrum $\times$ Total number of rulings on grating

1. Dependence on Number of ruled lines

From Equation (5) it is clear that R.P. is directly proportional to number of ruled lines i.e. it increases with increase in number of ruled lines.

2. Dependence on width or ruled surface

$$R = nN \left\{ \begin{aligned} (a + b) \sin \theta &= n\lambda \\ (a + b) \sin \theta &= \frac{N(a + b) \sin \theta}{N} = \frac{w \sin \theta}{\lambda} \end{aligned} \right.$$

$N(a + b) = w$ (Total width of ruled surface)
R.P. of a grating would not be affected if the number of lines N at a given width of ruled surface is changed.

Prob 3 What is X-ray diffraction? Deduce Bragg's Law for the diffraction of X-ray in a crystal, how Bragg's spectrometer is used to determine the wavelength of monochromatic X-rays. [R.T.U. 2016]

Sol. Diffraction of X-rays:

The diffraction of waves occurs when wavelength of waves happen to be comparable to the size of diffracting element. Visible light has wavelengths ranging from 4×10^{-7} m to 8×10^{-7} m and so it can easily be diffracted by diffraction grating having few thousand lines with a grating element (slit width) of same order as wavelength of visible light. X-rays are also electromagnetic waves but their wavelength is quite small ($\sim 1 \text{ \AA}$). Obviously, it is extremely difficult to make slits which are narrow enough for the study of X-ray diffraction. Since interatomic spacing in a crystal is of the same order as wavelength of X-rays, a crystal may therefore be used as a three dimensional diffraction grating. X-ray methods are advantageous because they are nondestructive. X-ray diffraction is very useful in many applications such as:

- It can be used to study crystal structure of an unknown material. This branch of science is called crystallography.

- It is used to measure the average spacing between layers or rows of atoms.

- It determines the orientation of a single crystal or grain.

- It measures the size, shape and internal stress of small crystalline regions.

Bragg's Law: Bragg's law refers to a simple equation derived by English physicists Sir W. H. Bragg and his son Sir W. L. Bragg in 1913. This equation explains why the faces of crystals appear to reflect (diffract) X-ray beams at certain angle of incidence θ . This observation is an example of X-ray wave interference, known as **X-ray diffraction (XRD)**. Bragg supposed that the incident waves undergo reflection at the various parallel planes of atoms in the crystal, with each plane reflecting only a small fraction of the radiation. The diffracted beams are found only when the reflection from various planes of atoms interfere constructively.

Consider a set of parallel atomic planes in a crystal separated by a distance d known as **interplanar spacing**. Suppose an X-ray beam is incident at a glancing angle θ (Fig. 1). It is scattered by the atoms like A and B in random directions. Constructive interference takes place only between those scattered waves which are reflected specularly (mirror like).

Fig. 1 shows two parallel rays R_1 and R_2 , reflected by two atoms A and B in two adjacent layers. The ray reflected from atom A goes to R_1' and from atom B goes to R_2' . Let us draw two normals AC and AD to R_2 and R_2' respectively. Each of these lines makes an angle θ with AB whose length is equal to the spacing d .

Then the path difference between the two reflected waves along R_1' and R_2' can be given as

$$\text{Path difference} = CB + BD \quad \dots (1)$$

$$\text{From } \Delta ACB \quad \sin \theta = \frac{CB}{AB}$$

$$CB = AB \sin \theta = d \sin \theta \quad \dots (2)$$

$$\text{Similarly, from } \Delta ADB \quad \sin \theta = \frac{BD}{AB}$$

$$BD = AB \sin \theta = d \sin \theta \quad \dots (3)$$

On substituting equation (2) and (3) in equation (1), we have

$$\text{Path difference} = d \sin \theta + d \sin \theta \quad \dots (4)$$

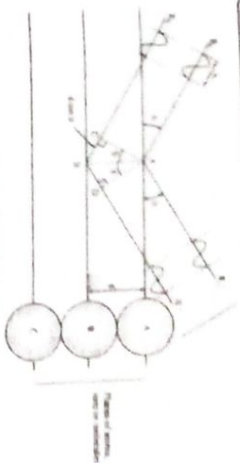


Fig. 1 : X-ray Diffraction

If this path difference is an integral multiple of wavelength, then constructive interference will occur between the reflected waves R_1' and R_2' , giving maximum intensity. Hence $2d \sin \theta = n\lambda$, $n = 1, 2, 3, \dots$ $\dots (5)$

Equation 5 is the well known **Bragg's equation** and is the mathematical expression of **Bragg's Law**.

Here d = lattice interplanar spacing of the crystal

θ = X-ray incidence angle (Bragg angle)

λ = wavelength of the characteristic X-rays.

It shows that for given value of λ and d , only for certain value of θ (corresponding to $n = 1, 2, 3, \dots$), the reflected waves add up in phase to give a strong reflected beam. The images obtained in these directions are the first, second third order diffraction images.

The diffracted beam fulfils the geometry of reflection but only for certain discrete values of θ for which Bragg's law is satisfied.

$$\text{We know that } (\sin \theta)_{\max} = 1 \quad \dots (6)$$

From equation (5) and (6), we have

$$\frac{n\lambda}{2d} \leq 1 \quad \dots (7)$$

Equation (7) is known as **Bragg condition** for X-ray diffraction, which shows that λ must not be greater than twice the interplanar spacing otherwise no diffraction will occur.

Bragg's X-Ray Spectrometer: It is an instrument invented by Bragg for the purpose of measuring the glancing angle and to determine the wavelength of an X-ray beam, by knowing the distance between the atomic planes of the crystal. The use of this spectrometer is explained by rotating crystal method discussed in next section.

Rotating Crystal Method (λ is fixed, θ is variable)

A beam of nearly parallel X-rays, by the slits S_1 and S_2 , are directed upon the face of the crystal (rock salt NaCl or calcite CaCO_3) mounted on the table of the spectrometer as shown in Fig. 2. After reflection from the crystal, the beam

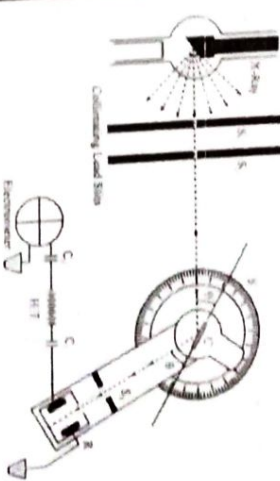


Fig. 2 : Variation of Intensity of Ionization Current with Glancing Angle for NaCl

If a graph is plotted between the intensity of ionization against the glancing angle known as ionization curve it appears as shown in Fig. 3. The peaks A_1 and B_1 correspond to two characteristic lines in the first order spectrum. Similarly A_2 , B_2 and A_3 , B_3 belong to the second and third order of spectrum. The maximum A_1 , A_2 and A_3 occur at angles 11.8° , 23.5° and 36° respectively. Now $\sin 11.8^\circ$, $\sin 23.5^\circ$, $\sin 36^\circ = 0.204$; 0.40 ; 0.63 which is nearly $1/5$; $2/5$; $3/5$. Thus, the Bragg's law is satisfied. The glancing angle θ is observed for different orders of spectra and wavelength λ is calculated from the Bragg's equation $2d \sin \theta = n\lambda$, knowing the lattice spacing ' d '.

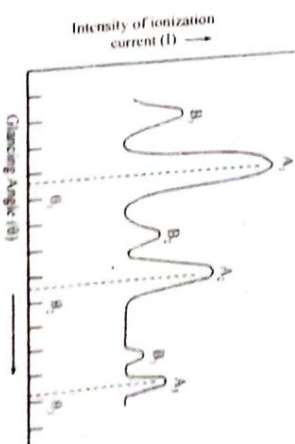


Fig. 3

Prob.4 (a) With schematic diagram, explain the working of a Michelson's Interferometer. Obtain the expression for radii of circular interference fringes. How shall you use to measure wavelength separation between two closed spaced spectral line?

(b) It is required to make an antireflection coating for light of wavelength $6000 \text{ \AA}$. If a thin film of $\mu = 1.25$ is to be coated on a glass plate of $\mu = 1.50$, what will be the minimum thickness of the film for normal incidence?

(c) Calculate the least width of a grating having 800 lines per cm to resolve the Sodium D - lines of wavelength $\lambda_1 = 5890 \text{ \AA}$ and $\lambda_2 = 5896 \text{ \AA}$. [R.T.U. 2017]

Sol.(a) Refer to Long Answer Prob. 1(a).

Sol.(b) Minimum thickness of the film for normal incidence;

$$t_{\min} = \frac{\lambda}{4\mu_f} = \frac{6000 \text{ \AA}}{4 \times 1.25} \\ = 1200 \text{ \AA}.$$

Sol.(c) We know that the number of slits on the grating = Width of the total number of lines $\times$ number of lines per cm

$$= x \times 800 \text{ [Here } x \text{ is the width]}$$

Given $\lambda_1 = 5890 \text{ \AA}$ and $\lambda_2 = 5896 \text{ \AA}$

So $d\lambda = 6 \text{ \AA}$ and

average $\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5890 + 5896}{2} \\ = 5893 \text{ \AA}$

Then $R_p = \frac{\lambda}{d\lambda} = \frac{5893}{6} \\ = 982.17 = 982$

Further $R_p = nN = 982$

or $2 \times N = 982$

or $N = \frac{982}{2} = 491$

To obtain least width

$$x \times 800 = 491$$

$$\Rightarrow x = \frac{491}{800} \text{ cm}$$

$$\Rightarrow x = 0.6137 \text{ cm.}$$

$$\Rightarrow x = 0.614 \text{ cm.}$$

Prob.5 In Bragg's reflection of X - rays, a reflection was found at the glancing angle of 30° with lattice planes of spacing $1.87 \text{ \AA}$. If this is a second order reflection, then calculate the wavelength of x-rays. [R.T.U. 2017]

Sol. According to Bragg's equation

$$2d \sin \theta = n \lambda$$

Here $\theta = 30^\circ$, $n = 2$ and $d = 1.87 \text{ \AA}$

$$\lambda = \frac{2d \sin \theta}{n} \\ = \frac{2 \times 1.87 \times \sin 30^\circ}{2} \\ = 1.87 \times \frac{1}{2}$$

$$\Rightarrow \lambda = 0.935 \text{ \AA}$$

So the wavelength of x-rays will be $0.935 \text{ \AA}$.

Prob.6(a) How shall you measure the wavelength separation of two closely spaced spectral lines e.g. D_1 and D_2 lines of Na? Derive the formula used.

(b) Newton's rings are formed using sodium light, between plano-convex lens and plane glass plate. What shall be the order of the ring which has twice the diameter of 20th dark ring?

[Note : Read 'lines' as 'lens'.]

(c) Two $\lambda/4$ thick layers are deposited on an ophthalmic glass ($\mu = 1.52$) to reduce reflection loss. The first layer has refractive index $\mu = 1.38$. Find the refractive index of the material of second layer. [R.T.U. Dec. 2016]

Sol.(a) Measurement of Difference in Wavelength of Sodium D_1 and D_2 Lines : For the measurement of difference in wavelength of two closely spaced sodium lines firstly the M_1 is adjusted for circular fringes.

For this mirror M_1 and M_2 should be perpendicular to each other and optical path G_1M_1 and G_2M_2' should be equal.

Let the source have two wavelengths λ_1 and λ_2 ($\lambda_1 > \lambda_2$), which are very close to each other.

Both the wavelength will form their separate fringe pattern but due to very small difference in wavelength both the pattern overlaps.

As the mirror M_1 is moved slightly the two patterns separated out and when the path difference is such that the dark fringe λ_1 falls on the bright rings of λ_2 , the result is maximum indistinctness. Now if mirror M_1 is further move, a condition comes when dark fringe of λ_1 and the dark rings of λ_2 (bright of λ_1 and λ_2) will overlap to each other. The result is maximum distinctness.

From these relation it is clear that first secondary maxima on either side of the central maxima have only about 4.5% of the height of the central maxima i.e. the most of the light is concentrated in the central maxima. It should be noted here that all the minima are equally spaced and their intensity is zero.

Sol.(b) Given : $\lambda_1 = 589.593 \text{ nm}$

$$\lambda_2 = 588.999 \text{ nm}$$

$$n = 1$$

$$N = ?$$

$$\lambda_{\text{mean}} = \frac{\lambda_1 + \lambda_2}{2} = \frac{589.593 + 588.999}{2}$$

$$= 589.296 \text{ nm}$$

We know that,

$$\frac{\lambda_{\text{mean}}}{\Delta\lambda} = nN$$

$$\text{Where, } \Delta\lambda = \lambda_1 - \lambda_2 = 0.594$$

$$N = \frac{\lambda_{\text{mean}}}{n \cdot \Delta\lambda}$$

$$= \frac{589.296}{1 \times 0.594}$$

$$N = 992.08 \approx 992$$

Prob.11(a) Explain, how does grating forms a spectrum of composite light falling normally on it.

(b) A diffraction grating used at normal incidence, gives green light of wavelength $\lambda_1 = 5400 \text{ \AA}$ in certain order coincident with violet line of wavelength $\lambda_2 = 4050 \text{ \AA}$ in next higher order. If angle of diffraction be 30° , how many lines are there per centimeter width of the grating.

[R.T.U. Dec. 2016]

Sol.(a) Formation of spectra with grating:

When a beam of light of wavelength λ , falls normally on a plane Transmission Grating the principle maxima formed in the direction of angle ' θ ', are given by grating equation

$$(a + b) \sin \theta = n\lambda, \text{ where } n = 0, 1, 2, \dots$$

Here $(a + b)$ = grating element having constant value for a given grating

θ = angle of diffraction.

n = order of spectrum

λ = wavelength of light

From the above equation it is clear that for a given order of spectrum the angle of diffraction θ varies with wavelength. Greater the angle of diffraction greater the wavelength (i.e.) ($\lambda_{\text{red}} > \lambda_{\text{violet}}$). So, $\theta_{\text{Red}} > \theta_{\text{Violet}}$.

Hence if the incident light is white then each order will contain principle maxima of different (colour) wavelength in different direction. The Principle maxima of all the wavelength (colour) corresponding to zero order spectrum for which ($n = 0$), will have the same direction $\theta = 0^\circ$ and combined to each other to form a zero order spectrum on screen. So, the colour of zero order spectrum will be same as that of incident light (white).

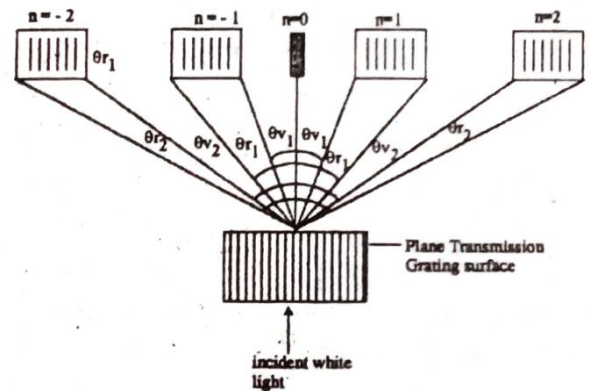


Fig.

But for higher order (i.e. for $n = 1, n = 2, \dots$) the angle of diffraction θ is different for each wavelength and found in different direction and these higher order spectrum is obtained on both the sides of central principle maxima, whose intensity is the maxima.

In result we find a central bright fringe of white colour for ($n = 0$) and in other order ($n = 1, 2, \dots$) the violet colour is found in inner most position whereas red colour is in outermost position since ($\theta_v < \theta_r$).

Sol.(b) We know that, $(a + b) \sin \theta = n\lambda$

Let n^{th} order maxima of λ_1 coincide with $(n + 1)^{\text{th}}$ order maxima of λ_2 , then

$$(a + b) \sin \theta = n\lambda_1 \text{ and } (a + b) \sin \theta = (n + 1)\lambda_2$$

From the given condition, the line is coincident.

$$\therefore n\lambda_1 = (n + 1)\lambda_2$$

$$\text{or } n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$\text{Now, } (a + b) \sin \theta = n\lambda_1 = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

$$\text{No. of lines per cm.} = \frac{1}{a + b} = \frac{\sin \theta (\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$$

From the expression of I_0, I_1, I_2 , it is apparent that most of the incident light is concentrated in the principal maximum.

Intensity Distribution Graph : The variation of intensity with α is shown in fig.

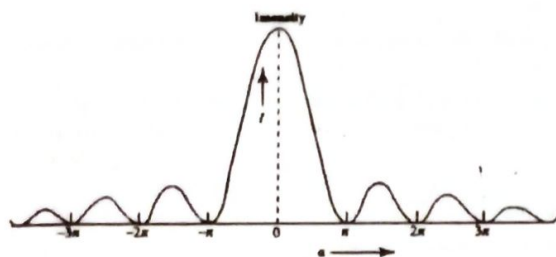


Fig.3 : Intensity I vs α

As shown in the fig. above, the diffraction pattern comprises of a central principal maxima occurring in the direction of incident rays. There are secondary maxima of decreasing intensity found on either sides of it at positions $\alpha = \pm 3\pi/2, \pm 5\pi/2$, and so on. Between secondary maxima, there are minima at positions $\alpha = \pm \pi, \pm 2\pi, \pm 3\pi, \dots$. It is found that these secondary maxima do not fall exactly midway between the two minimum but they are displaced towards the centre of the pattern. The displacement decreases as the order of maximum increases.

Prob.18 Michelson interferometer experiment is performed with a source which have two wavelengths $4882\text{\AA}$ and $4886\text{\AA}$. By what distance does the mirror have to be moved between two positions of disappearance of fringes?

[R.T.U. 2012]

Sol. Given : $\lambda_1 = 4882\text{\AA} = 4882 \times 10^{-10} \text{ m}$

$\lambda_2 = 4886\text{\AA} = 4886 \times 10^{-10} \text{ m}$

In Michelson interferometer wavelength difference is given by-

$$\lambda_2 - \lambda_1 = \frac{\lambda_{av}^2}{2d}$$

$$d = \frac{\lambda_{av}^2}{2(\lambda_2 - \lambda_1)}$$

$$\lambda_{av} = \frac{\lambda_1 + \lambda_2}{2} = \frac{4882 + 4886}{2} = 4884\text{\AA}$$

$$\lambda_{av} = 4884 \times 10^{-10} \text{ m}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = 4886 - 4882 = 4\text{\AA} = 4 \times 10^{-10} \text{ m}$$

$$d = \frac{(4884 \times 10^{-10})^2}{2 \times 4 \times 10^{-10}} = 0.298 \text{ mm}$$

Prob.19 In a Michelson interferometer, when 100 fringes were shifted, the final reading of the screw was found to be 10.735 mm . If the wavelength of the light was $5.92 \times 10^{-7} \text{ m}$, what was the initial reading of the screw?

[R.T.U. 2012]

Sol. Given

$$x_2 = 10.735 \text{ mm}$$

$$\lambda = 5.92 \times 10^{-7}$$

$$m = 5.92 \times 10^{-4} \text{ mm}$$

$$N = 100 \text{ fringe}$$

$$x_1 = ?$$

We know that

$$\lambda = \frac{2x}{N}$$

$$x = \frac{\lambda N}{2} = x_2 - x_1 = \frac{\lambda N}{2}$$

$$\Rightarrow 10.735 - x_1 = \frac{5.92 \times 10^{-4} \times 100}{2}$$

$$10.735 - x_1 = 0.0296$$

$$\Rightarrow x_1 = 10.735 - 0.0296 = 10.7054 \text{ mm}$$

Prob.20(i) The width of a slit is 0.012 mm . Monochromatic light is incident on it. The angular position of first bright line is 5.2° . Calculate the wavelength of incident light.

(ii) What is difference in Fresnel's and Fraunhofer diffraction?

[R.T.U. 2011]

Sol.(i) Given width of slit (a) = $0.012 \text{ mm} = 1.2 \times 10^{-5} \text{ meter}$

Angular position of first bright line $\theta = 5.2^\circ$

$$= \frac{5.2 \times \pi}{180} \text{ Radian}$$

Wavelength of light = ?

The condition for bright is

$$a \sin \theta = \left(n + \frac{1}{2} \right) \lambda$$

For first bright $n = 1$

$$a \sin \theta = \left(1 + \frac{1}{2} \right) \lambda$$

$$a \sin \theta = 3\lambda / 2$$

$$\lambda = \frac{2a \sin \theta}{3} = \frac{2a \theta}{3}$$

$$\left[\begin{array}{l} \text{Since for small values of angle} \\ \sin \theta = \theta \end{array} \right]$$

$$\lambda = \frac{2 \times 1.2 \times 10^{-5} \times 5.2 \times 3.14}{3 \times 180}$$

$$\lambda = 7.257 \times 10^{-7} \text{ meter}$$

$$\lambda = 7257 \text{ \AA}$$

Sol.(ii) When slit width is large compared to wavelength the diffraction is limited to boundary only.

Some of the main difference between Fresnel's and Fraunhofer's class of diffraction are listed below :

	Fraunhofer's diffraction	Fresnel's diffraction
1.	The distance of source and screen are effectively ∞ (infinite) from diffracting element.	The source and screen are at finite distance from diffracting element.
2.	To achieve ∞ (infinite) distance, lens are used.	No such lens is used.
3.	The incident and diffracted wavefront is plane.	The incident and diffracted wavefront are generally spherical or cylindrical.
4.	It is a limiting case when separation becomes ∞ (infinite).	It is a general case.
5.	The effect of several diffracting elements can be added together.	No such addition of effect is possible.
6.	Theoretical treatment is much simple.	Theoretical treatment is complex.
7.	The angular inclinations are important.	Distances are important in this case.

$$\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\frac{d^2 \psi(x)}{dx^2} + \frac{2m}{\hbar^2} (E - V(x))\psi(x) = 0$$

(iv) Time independent three dimension Schrodinger's wave equation

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x) + V(x)\psi(x) = E\psi(x)$$

For free particle $V(x) = 0$

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x) = E\psi(x)$$

□ **Physical Significance of Wave Function (ψ):** Wave function is a complex quantity, which itself has no physical significance. But square of its magnitude gives the probability of finding a particle in a given volume of space at a given time.

Motion of a free particle in 1 dimension box

$$\text{Wave function } \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$$

a = width of box, $n = 1, 2, 3, \dots$

PREVIOUS YEARS QUESTIONS

I. SHORT ANSWER TYPE QUESTIONS

Prob.1 What do you understand by wave function?

[R.T.U. 2017]

OR

What is wave function?

Sol. The varying quantity characterising the matter wave is denoted by $\psi(x, y, z)$ at a point i.e. the quantity whose variation makes up the matter wave is known as wave function or wave amplitude.

Prob.2 Give the normalization condition for the wave function.

Sol. If the wave function $\psi_m(r, t)$ is multiplied with its complex conjugate $\psi_m^*(r, t)$ and integrate it over the whole volume is equal to unity i.e.

$$\int_{-\infty}^{\infty} \psi_m^*(r, t) \psi_m(r, t) dV = 1$$

Energy eigen value

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$

□ **Probability of Finding the Particle in Given Limits**
The probability between the given limit x_1 and x_2 is given as (in one dimension case)

$$P = \int_{x_1}^{x_2} |\psi(x, t)|^2 dx$$

□ **Essential Boundary Conditions for Wave Function**

The wave function ψ should essentially satisfy the following boundary conditions:

- (i) ψ must be finite for all values of (x, y, z) because infinite value of ψ means infinite probability of finding the particle, which is not true due to the uncertainty principle.
- (ii) ψ must be well behaved means single valued because multiple value of ψ will give multi-probability, which is not true.

(iii) ψ and its derivatives $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous and differentiable since derivative of a discontinuous function is always infinite means infinite value of ψ which is incorrect.

(iv) ψ should follow the superposition principle.

This condition is called **normalization condition** for the wave function.

Prob.3 Give the orthogonalization condition for the wave function.

Sol. If the wave function $\psi_n(r, t)$ is multiplied with other complex conjugate $\psi_n^*(r, t)$ and integrate it over whole volume vanishes i.e.

$$\int_{-\infty}^{\infty} \psi_m^*(r, t) \psi_n(r, t) dV = 0$$

Here $n \neq m$
Then this condition is called **orthogonal condition** of wave function.

Prob.4 An eigen function of an operator $\frac{d^2}{dx^2}$ is $\psi = e^{ikx}$. Find corresponding eigen value.

Sol. Given: $\psi = e^{ikx}$

$$\text{Let operator } A = \frac{d^2}{dx^2}$$

$$A\psi = \frac{d^2}{dx^2} e^{ikx}$$

$$A\psi = \frac{d}{dx} \left(\frac{d}{dx} e^{ikx} \right)$$

$$A\psi = \frac{d}{dx} (ik e^{ikx})$$

$$A\psi = ik^2 e^{ikx}$$

since

$$\psi = e^{ikx}$$

$$A\psi = k^2 \psi$$

So, eigen value for operator A is k^2 .

Prob.5 Determine the expectation value of position of a particle in one dimensional box.

Sol. Eigen function of a particle in 1D box is:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$$

$\therefore$ Expectation value of position of a particle is defined as

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi_n^*(x) x \psi_n(x) dx$$

$$\langle x \rangle = \left(\frac{2}{a} \right) \int_0^a x \sin^2\left(\frac{n\pi}{a}x\right) dx$$

$$\langle x \rangle = \frac{2}{a} \cdot \frac{a^2}{4} = \frac{a}{2}$$

Expectation value of position of a particle in 1D box is $a/2$.

II. LONG ANSWER QUESTIONS

Prob.1 Write down the Schrodinger's time independent wave equation for a free particle confined in one-dimensional box of size a . Obtain eigen values and normalized wave functions for this particle.

[R.T.U. 2018, 2016, (I Sem), 2008]

OR

Write down Schrodinger wave equation for a particle enclosed in one dimensional box of size a . Solve it to get eigenvalues and eigenfunctions.

[R.T.U. 2015, 2012, Rd.Um, 2005]

OR

Solve Schrodinger wave equation for a particle in one-dimensional box. Discuss the various energy states.

OR

Solve the Schrodinger wave equation for a particle trapped in one dimensional box of side a and obtain its energy eigen values and eigen functions.

[R.T.U. 2012]

OR

Write down Schrodinger's equation for a particle confined in a one dimensional box. Obtain the wave function for a particle confined in this box.

[R.T.U. 2011]

Sol. Particle in One Dimensional Box (1 - D box)

In quantum mechanics a one dimensional box means a system in which a particle is constrained to move in one direction in a box of potential $V = 0$. The limits of the potential function for the particle is defined as follows:

$$V(x) = 0, \quad 0 < x < a$$

$$V(x) = \infty, \quad x \leq 0$$

$$V(x) = \infty, \quad x \geq a$$

...(1)

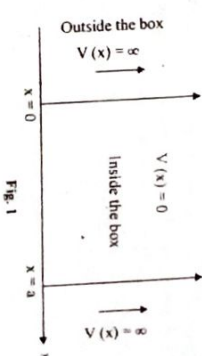


Fig. 1

Suppose a particle of mass m is in motion along the x -axis (Fig. 1). Let no other force act on this particle, so that the potential energy of the particle is constant. For convenience, the constant potential energy is taken to be zero. The one dimensional, time independent wave equation under this condition is given by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \dots (2)$$

where $V(x) = 0$, so from equation (2), we have

$$\frac{d^2 \psi}{dx^2} + \left[\frac{2m}{\hbar^2} E \right] \psi = 0 \quad \dots (3)$$

Here $E = \frac{p_x^2}{2m}$ where p_x is momentum of the particle. i.e., $\frac{d^2 \psi}{dx^2} + k^2 \psi = 0 \quad \dots (4)$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \quad \dots (5)$$

$$E = \frac{\hbar^2 k^2}{2m} \quad \dots (6)$$

For a particular value of the energy E , k is a constant. The general solution of equation (5) is of the form

$$\psi(x) = A \sin kx + B \cos kx \quad \dots (7)$$

where A and B are arbitrary constants. The values of these constants can be obtained by applying the boundary conditions of the problem.

$$\text{(i) At } x=0, \psi(x)=0$$

$$\text{From this we get } 0 = A \sin k \cdot 0 + B \cos k \cdot 0$$

$$\text{hence } \psi(x) = A \sin kx \quad \dots (8)$$

$$\text{(ii) At } x=a, \psi(x)=0$$

Now using the second condition, we have

$$\psi(a) = A \sin ka = 0 \quad \dots (9)$$

We cannot take $A = 0$, because it can not be solved. Hence equation (9) is explained as follows

$$ka = n\pi$$

$$\text{or } k = \frac{n\pi}{a} \quad \dots (10)$$

$$\text{where } n = 1, 2, 3, \dots$$

Here n should not be zero because if we consider $n=0$ then the energy eigen value vanishes in the 1-D box, since in this case $k=0$, $E=0$.

Hence, the wave functions for the motion of the particle in the region $0 < x < a$ are given by

$$\psi_n(x) = A \sin \frac{n\pi x}{a}$$

$$\psi_n = A \sin \frac{n\pi x}{a} \quad \dots (11)$$

Equation (11) is known as eigen function for a particle in one dimensional box.

(A) Eigen value of the Wave Function

On substituting the value of k from equation (10) in equation (6), we get

$$E = \frac{\hbar^2 k^2}{2m} = \left[\frac{n^2 \pi^2}{a^2} \right] \left[\frac{\hbar^2}{8\pi^2 m} \right]$$

$$E_n = \frac{n^2 \hbar^2}{8ma^2} = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \quad \dots (12)$$

Equation (12) reveals that the lowest energy of the particle is obtained by putting $n=1$ in equation (12) and it is

$$E_1 = \frac{\hbar^2}{8ma^2} = \frac{\pi^2 \hbar^2}{2ma^2} \text{ and } E_n = n^2 E_1 \quad \dots (13)$$

This is known as zero point energy of the system.

Engineering Physics

The wave-function ψ , corresponding to E_n , are called eigen functions of the particle. The integer n corresponding to the energy E_n is called the quantum number of the energy level E_n . Fig. 2 shows the schematic representation of energy level for a particle in one dimensional box.

(B) Calculation of Constant A (Normalization of the Eigen Function) : Since the solutions of the wave equation are independent of the magnitude of A this permits us to choose a value for A in order to satisfy any condition we want to impose. The total probability that the particle is somewhere in the box must be unity. Using normalization condition, we have

$$\int_0^a P_n dx = \int_0^a |\psi_n|^2 dx = 1$$

$$\int_0^a A^2 \sin^2 \frac{n\pi x}{a} dx = 1 \quad \dots (14)$$

$$A^2 \left[\frac{x}{2} - \frac{a}{4\pi n} \sin \frac{2n\pi x}{a} \right]_0^a = 1$$

$$\left(\frac{A}{2} \right)^2 \left[x - \frac{a}{2\pi n} \sin \frac{2n\pi x}{a} \right]_0^a = 1$$

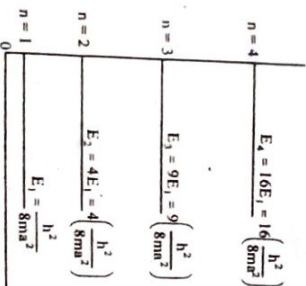


Fig. 2 : Energy level diagram for a particle in 1-D box

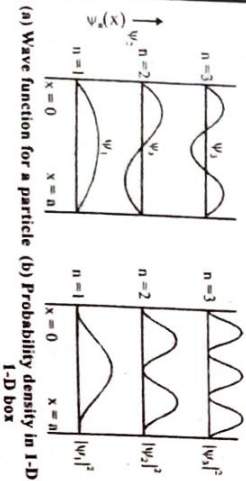


Fig. 3

Engineering Physics

The second term of the integrated expression becomes zero at both the limits.

Thus

$$\frac{A^2 a}{2} = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{a}} \quad \dots (15)$$

the normalized wave function is

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \quad \dots (16)$$

The wave functions for the first three values of n are shown in fig. 3(a). It is evident from fig. 3(a) that the wave function ψ_1 has two nodes at $x=0$ and $x=a$. It is also clear that the wave function ψ_2 has three nodes at $x=0$, $x=\frac{a}{2}$ and $x=a$, and the wave function ψ_3 has four nodes at $x=0$, $x=\frac{a}{3}$, $x=\frac{2a}{3}$ and at $x=a$. Thus the wave function ψ_n will have $(n+1)$ nodes. The probability density function, for the particle in one dimensional box are represented as shown in fig. 3(b).

Problem 2(a) Derive Schrödinger's time dependent equation. Explain the following :

- Hamiltonian,
 - Physical significance of wave function, and
 - Normalized and orthogonal wave functions.
- (b) x - rays with $\lambda = 1\text{Å}$ are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident beam.

(i) What is the Compton shift $\Delta\lambda$?

(ii) What kinetic energy is imparted to the recoil electron?

[K.T.U. 2017]

Sol. (a) (i) Time Dependent Schrödinger's Wave Equation : Let a particle of mass m is moving along x -direction in conservative force field. If the potential energy, momentum and total energy of the particle is V , p_x (along x -direction) and E respectively then the wave function for the moving particle can be given as :

$$\psi(x,t) = A \exp \left\{ \frac{-i}{\hbar} (Et - p_x x) \right\} \quad \dots (1)$$

On differentiating equation (1) with respect to time, we get

$$\frac{\partial \psi(x,t)}{\partial t} = \left(\frac{-i}{\hbar} E \right) A \exp \left\{ \frac{-i}{\hbar} (Et - p_x x) \right\}$$

$$\frac{\partial \psi(x,t)}{\partial t} = \frac{-i}{\hbar} E \psi(x,t)$$

$$E \psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t} \quad \dots (2)$$

Here $\left[E \rightarrow i\hbar \frac{\partial}{\partial t} \right]$ is known as Total Energy Operator

On differentiating equation (1) with respect to position x , we get

$$\frac{\partial \psi(x,t)}{\partial x} = \frac{-i}{\hbar} (-p_x) A \exp \left\{ \frac{-i}{\hbar} (Et - p_x x) \right\}$$

$$\frac{\partial \psi(x,t)}{\partial x} = \frac{i}{\hbar} p_x \psi(x,t) \quad \dots (3)$$

$$p_x \psi(x,t) = \hbar \frac{\partial \psi(x,t)}{\partial x} = -i\hbar \frac{\partial \psi(x,t)}{\partial x} \quad \dots (4)$$

Here $\left[p_x \rightarrow -i\hbar \frac{\partial}{\partial x} \right]$ is known as Momentum Operator

in x -direction.

On differentiating equation (3) with respect to position x , we get

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \left(\frac{i}{\hbar} p_x \right) \left(\frac{i}{\hbar} p_x \psi(x,t) \right)$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \left(-\frac{p_x^2}{\hbar^2} \right) \psi(x,t)$$

$$p_x^2 \psi(x,t) = -\hbar^2 \frac{\partial^2 \psi(x,t)}{\partial x^2} \quad \dots (5)$$

By means of the principle of conservation of energy in non relativistic mechanics, we have

Total energy = Kinetic energy + Potential energy

$$E = \frac{p_x^2}{2m} + V \quad \dots (6)$$

On multiplying by $\psi(x,t)$ in equation (6) from left, on both the sides

$$E \psi(x,t) = \frac{p_x^2}{2m} \psi(x,t) + V \psi(x,t) \quad \dots (7)$$

Using equation (2) and (5) in equation (7), we get

$$\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V \psi(x,t) \quad \dots (8)$$

Equation (8) is known as the *Schrödinger one dimensional (1D) time dependent wave equation*.

For free particle $V = 0$ (the potential energy is considered to be zero).

Equation (8) reduces to

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} \quad \dots(9)$$

Equation (9) is known as *Schrödinger one dimension (1D) time dependent wave equation for free particle*.

A free particle is a particle which does not bound by an external force or in field free space. In quantum mechanics, it means a region of constant potential, usually taken equal to zero in the region of interest since potential can be arbitrary set to zero at any point in space.

In three dimensional case - on generalizing equation (8), we get

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \frac{-\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(r,t) + V \psi(r,t) \quad \dots(10)$$

$$\text{Here } \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2 \quad \dots(11)$$

known as *Laplacian operator*.

From equation (10) and (11), we have

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi(r,t) + V \psi(r,t) \quad \dots(12)$$

Equation (12) known as the *Schrödinger three dimensional (3D) time dependent wave equation*.

For free particle $V = 0$, equation (12) reduces to

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi(r,t) \quad \dots(13)$$

Equation (13) known as *Schrödinger three dimensional (3D) time dependent wave equation for free particle*.

On rewriting equation (12), we get

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi(r,t) = i\hbar \frac{\partial \psi(r,t)}{\partial t} \quad \dots(14)$$

Here the expression $\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right)$ is called

Hamiltonian operator (H).

$$H = \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \quad \dots(15)$$

From equation (14) and (15), we can write

$$H \psi = i\hbar \frac{\partial \psi}{\partial t} \quad \left(\text{Since } E \rightarrow i\hbar \frac{\partial}{\partial t} \right)$$

$$H \psi = E \psi \quad \dots(16)$$

This equation is called *Hamiltonian form of the Schrödinger wave equation*.

(ii) **Physical Significance of Wave Function $\Psi(\vec{r}, t)$:** As wave function is a complex quantity, so it has no physical meaning and can be expressed in form of complex quantity

$$\psi(x, y, z, t) = A + iB \quad \dots(1)$$

where A and B are real functions of variable (x, y, z, t) .

The complex conjugate of equation (1) can be given as

$$\psi^*(x, y, z, t) = A - iB \quad \dots(2)$$

on multiplying equation (1) and (2), we get

$$= \psi(x, y, z, t) \psi^*(x, y, z, t) = A^2 + B^2$$

$$P = |\psi(x, y, z, t)|^2 = A^2 + B^2 \quad \dots(3)$$

$$P = |\psi(x, y, z, t)|^2 = \psi(x, y, z, t) \psi^*(x, y, z, t) \quad \dots(4)$$

The product of these two complex quantity, when square root is eliminated i.e. $|\psi(x, y, z, t)|$ is called the *modulus of waveflux* ψ . The quantity $|\psi(x, y, z, t)|^2$ is called the *probability density*. **Maxborn** was the first who explain the wave function as follows- According to him the probability that the particle will be found in a defined volume element dV surrounding the particle at position (x, y, z) at any time t is given by

$$P dV = \psi(x, y, z, t) \psi^*(x, y, z, t) dV$$

$$= |\psi(x, y, z, t)|^2 dV \quad \dots(5)$$

Thus the probability to find the particle in a defined space is directly proportional to integral $|\psi(x, t)|^2$ over the volume of space.

If the motion of the particle is in one-dimension, then

$$P dx = \psi(x, t) \psi^*(x, t) dx = |\psi(x, t)|^2 dx \quad \dots(6)$$

Then the total probability of finding the particle over all space in 1-D will be

$$P = \int_{-\infty}^{+\infty} |\psi(x, t)|^2 dx \quad \dots(7)$$

For general case in 3-D space, we observe

$$P = \int_{-\infty}^{+\infty} |\psi(x, y, z, t)|^2 dV \quad (dV = dx dy dz) \quad \dots(8)$$

This equation must be finite in order to represent a real particle.

(iii) **Normalization Condition :**

$$\int_{\text{All space}} \psi^* \psi dx dy dz = \int_{\text{All space}} |\psi|^2 dx dy dz = 1$$

This is called normalization condition. The wave function which satisfies above equation is said to be normalized to unity.

Orthogonal Wave Function

Let y_i and y_j are two different wave functions then if

$$\int_{\text{All space}} \psi_i \psi_j^* dV = 0 \text{ or } \int_{\text{All space}} \psi_j \psi_i^* dV = 0; i \neq j$$

Then these functions are said to be mutually orthogonal. This is known as orthogonality condition.

Sol.(b) Given : X-ray wavelength $\lambda = 1 \text{ \AA}$

Scattering angle $\theta = 90^\circ$

(i) **Compton shift**

$$\Delta \lambda = \lambda' - \lambda = 0.0243(1 - \cos \theta) \text{ \AA}$$

$$\Delta \lambda = 0.0243(1 - \cos 90^\circ) = 0.0243 \text{ \AA}$$

$$\lambda' - \lambda = 0.0243 \text{ \AA}$$

$$\lambda' = \lambda + 0.0243 = 1 + 0.0243 = 1.0243 \text{ \AA}$$

(ii) **Kinetic energy of recoil electron :**

$$K.E. = h\nu - h\nu' = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right)$$

$$K.E. = \frac{hc(\lambda' - \lambda)}{\lambda' \lambda} = \frac{hc \Delta \lambda}{\lambda' \lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 0.0243 \times 10^{-10}}{1 \times 10^{-10} \times 1.0243 \times 10^{-10}}$$

$$K.E. = 4.72 \times 10^{-17} \text{ Joule}$$

$$K.E. = \frac{4.72 \times 10^{-17}}{1.6 \times 10^{-19}} = 295 eV$$

Prob.3 (a) Obtain an expression for shift in wavelength of scattered photon by Compton scattering and show that Compton shift depends only on scattering angle. What is Compton wavelength?
(b) Derive one dimensional time dependent Schrodinger's wave equation and also explain the physical significance of wave function.

$$c = 3 \times 10^8 \text{ m/s}$$

$$\Delta\lambda = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} (1 - \cos 90^\circ)$$

$$\Delta\lambda = 0.0242 (1 - \cos 90^\circ)$$

$$\Delta\lambda = 0.0242 \text{ \AA}$$

$$\lambda' = \lambda + \Delta\lambda \\ = 1.0 + 0.0242 = 1.0242 \text{ \AA}$$

Prob.9 Find the lowest energy of an electron confined to move in a one dimensional box of length 1 \AA.

[R.T.U. 2012]

Sol. We have the relation

$$E_n = n^2 \frac{\pi^2 h^2}{2ma^2}$$

The lowest energy of (an electron) the particle obtained by putting $n = 1$.

The energy is given by

$$E_1 = \frac{\pi^2 h^2}{2ma^2}$$

$$h = 10^{-34} \text{ J-s}$$

$$a = 1 \text{ \AA} = 10^{-10} \text{ m}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$E_1 = \frac{(3.14)^2 \times (10^{-34})^2}{2 \times (9.1 \times 10^{-31}) (10^{-10})^2}$$

$$= 0.5411 \times 10^{-17}$$

$$= 5.411 \times 10^{-18} \text{ J}$$

$$E_1 = \frac{5.411 \times 10^{-18}}{1.602 \times 10^{-19}} = 33.78 \text{ eV}$$

Prob.10 (i) A particle is moving in one dimensional potential box (of infinite height) of width 25 \AA. Calculate the probability of finding the particle within an interval of 5 \AA at the center of the box when it is in its state of least energy.

(ii) Explain normalized and orthogonal wave functions.

[R.T.U. 2011]

Sol. (i) The wave function of a particle enclosed with an infinite potential well is given by

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}\right)x$$

When the particle is in the least energy state $n = 1$, hence in this case

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$$

At the centre of the box $x = a/2$. The probability of finding the particle in the unit interval at the centre of box is given by

$$|\psi(x)|^2 = \left[\sqrt{\frac{2}{a}} \sin\left(\frac{\pi(a/2)}{a}\right) \right]^2 = \frac{2}{a} \sin^2 \frac{\pi}{2} = \frac{2}{a}$$

the probability P in the interval Δx is given by

$$P = |\psi(x)|^2 \Delta x = \frac{2}{a} \Delta x$$

$$\text{Given: } a = 25 \text{ \AA} = 25 \times 10^{-10} \text{ m}$$

$$\text{and } \Delta x = 5 \text{ \AA} = 5 \times 10^{-10} \text{ m}$$

$$\therefore P = \frac{2 \times 5 \times 10^{-10}}{25 \times 10^{-10}} = 0.4$$

Sol. (ii) Normalized and Orthogonal Wave Functions :
Refer to Long Answer Prob.2(a)(iii).

Prob.11 Calculate the smallest possible uncertainty in position of a electron moving with velocity $3 \times 10^7 \text{ m/s}$.
[R.T.U. 2010]

Sol. Given $v = 3 \times 10^7 \text{ m/sec}$

Smallest possible uncertainty in electrons position

$$\Delta x + \Delta p_x = h$$

$$\Delta x \cdot m \cdot \Delta v = h$$

$$\Delta x = \frac{h}{mv}$$

$$\Delta x = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^7}$$

$$\Delta x = 0.24 \times 10^{-10} \text{ meter} = 0.24 \text{ \AA}$$

Prob.12 Write short notes on :

(i) Wave Particle Duality

(ii) Matter Waves

Sol.(i) Wave Particle Duality : Wave-particle duality is the concept in quantum mechanics that energy-carrying waves can also behave like particles and that particles can also display a wave aspect. Light, for example, demonstrates the wave phenomena of diffraction and interference but, under other circumstances, appears to be a stream of tiny particles called photons. Electrons, on the other hand, which normally

behave like particles, can be made to diffract and interfere as if they consisted of waves. Thus scientists began to conclude that light has both wave and particle nature.

In 1924, a French physicist, Louis de Broglie came up with a theory. He suggested that all particles in this universe is associated with wave nature also, i.e. everything in this world be it a small photon or a giant elephant, everything has an associated wave with itself, it is a different matter that the wave nature is noticeable or not. All matter can exhibit wave-like behaviour. For example, a beam of electrons can be diffracted just like a beam of light or a water wave. Matter waves are a central part of the theory of quantum mechanics, an example of wave-particle duality. The concept that matter behaves like a wave is also referred to as the de Broglie hypothesis due to having been proposed by Louis de Broglie. (ii) **Matter Waves :** According to De-Broglie hypothesis, particles like electron, proton, neutron etc show wavelike properties associated with them. 'All material particles in motion possess a wave character'. These wave are called 'Matter Waves' or 'De-Broglie Waves'.

A light wave of frequency ν is associated with a photon of energy E is given by

$$E = h\nu \quad \dots(1)$$

According to Einstein theory of relativity a mass m is equivalent to an amount of energy E according to the relation

$$E = mc^2$$

$$m = \frac{E}{c^2} \quad \dots(2)$$

where c is the velocity of light in free space.

Hence the mass of the photon.

$$m = \frac{h\nu}{c^2} \quad \dots(3)$$

The photon travels with the velocity of light c . Therefore, the momentum of the photon is given by

$$p = mc = \frac{h\nu}{c^2} c = \frac{h\nu}{c} \quad \dots(4)$$

But $\nu = \frac{c}{\lambda}$, where λ is the wavelength of light.

$$p = \frac{h}{\lambda} \quad \dots(5)$$

$$\lambda = \frac{h}{p} \quad \dots(6)$$

thus, the wavelength associated with a photon is h/p . A particle of (relativistic) mass m and moving with a

velocity v has both wave and particle nature.

Prob.13

Sol. It is periodic sound wave in one of matter

wave is whose wave function

wave function of molecule

L the x-direction

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it cannot physical

CHAPTER IN A NUTSHELL

□ **Coherence** : The waves maintaining constant and predictable phase difference are called coherent waves. There are two types of coherence :

(a) **Temporal Coherence** : If the phase difference measured at a single point in space at beginning and at end of a time interval Δt remains predictable and constant then the waves are said to temporal coherence.

(b) **Spatial Coherence** : The spatial coherence is the definite phase relationship between the waves at different points in space.

$$\text{Spatial coherence } L_w = \frac{\lambda d}{a} \text{ for slits}$$

d = distance between slits

a = width of the slit.

□ **Coherence Length** : The average length of the wavetrains for which the field remains sinusoidal is called 'Coherence Length'. It is denoted by ℓ_c .

$$\ell_c = c\tau_c$$

Temporal coherence is determined by the coherence length ℓ_c which depends on the line width $\Delta\lambda$ of central wavelength λ .

□ **Coherence Time** : Coherence time is defined as the

average time interval during which the field remains sinusoidal and the phase of the wavetrain can be predicted reliably. It is denoted by τ_c .

$$\tau_c = \frac{\ell_c}{c} = \frac{1}{\Delta\nu}$$

□ **Optical Fiber** : According to Snell's law Optical Fiber propagates light using the phenomena of total internal reflection

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_2}$$

Here angle i and angle r are the critical angles for core (medium-1) and cladding (medium-2) with the normal and n_1 and n_2 are refractive indices of core and cladding respectively.

□ **Numerical Aperture** : It is defined as the sine of angle of acceptance.

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2}$$

$$\text{N.A.} = n_o \sin i_m$$

i_m = Maximum acceptance angle

n_o = Refractive index of medium from which light is launched to fiber.

PREVIOUS YEARS QUESTIONS

I. SHORT ANSWER TYPE QUESTIONS

Prob.1 What is the physical meaning of numerical aperture of an optical fiber?
[R.T.U. 2018]

Sol. Numerical aperture is the light gathering capacity of an optical fiber and is proportional to acceptance angle. Numerically it is equal to sine of acceptance angle.

$$\text{N.A.} = \sin \theta_A = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0}$$

If the refractive index of air medium $\mu_a = 1$ then

$$N.A. = \sin i_a = \sqrt{1^2 - \mu_a^2}$$

Prob.3 What are coherent sources? How are they obtained in practice? (J.T.E. 2007)

Sol. Two light sources S_1 and S_2 are said to be coherent if they emit light waves of same frequency, almost same amplitude and having a constant or zero phase difference between them. It should be noted here that 'coherent' doesn't mean 'in same phase'. It only means 'having constant phase relationship'.

Coherent sources can be produced if both the source of light S_1 and S_2 are obtained from a single source S by any means, then all the random phase changes occurring in source S simultaneously accompanied by the same change in source S_1 and S_2 , so the relative phase remains constant. These sources S_1 and S_2 are called **coherent sources**.

Prob.4 Discuss the attenuation and dispersion of signals in optical fibre. (J.T.E. 2007)

Sol. As waves propagate in optical fibre, they may become weaker due to various reasons. The amplitude of waves may decrease. This is known as **attenuation**. Low attenuation is useful for long distance communication fibres since amplification of the signal is expensive and complicated.

Prob.5 What do you understand by temporal coherence?

Sol. Temporal coherence is a measure of the correlation between the phases of a light wave at different points along the direction of propagation. It is the correlation between the wave at one place at different times along the path of a monochromatic source. It is related to the coherence time.

Prob.6 What do you understand by spatial coherence?

Sol. Spatial coherence is a measure of the correlation between the phases of a light wave at different points transverse to the direction of propagation.

Prob.7 Define the coherence time.

Sol. Coherence time is defined as the average time interval during which the field remains unimodulated and the phase of the wave can be predicted reliably.

Prob.7 Give any two applications of optical fibre.

Sol. (i) Optical fibres are extensively used as information transmitting media due to its low loss property over wide range of frequencies.

(ii) It is used in communication due to very less attenuation in signal amplitude over a long distance. Telephone companies use optical fibre throughout their system as the backbone architecture and as the long distance connection between city-phone systems.

Prob.8 An optical fibre has refractive index of core to be 1.5 and the refractive index of cladding to be 0.01. Determine numerical aperture and maximum angle of acceptance. (J.T.E. 2004)

Sol. Given: $\mu_{\text{core}} = 1.5$

$$\mu_{\text{clad}} = 0.01$$

$$\text{Then } \mu_{\text{rel}} = 1.50 - 0.01 = 1.49$$

The numerical aperture

$$N.A. = \sqrt{(\mu_{\text{core}}^2 - \mu_{\text{clad}}^2)} = \sqrt{(1.5^2 - 0.01^2)} = 0.173$$

Now acceptance angle

$$\theta = \sin^{-1}(N.A.) = \sin^{-1}(0.173) = 10^\circ$$

II. LONG ANSWER TYPE QUESTIONS

Prob.1 (a) What do you mean by the word coherence? Explain temporal and spatial coherence. Give example of one experiment each which demonstrates temporal and spatial coherence. (J.T.E. 2004)

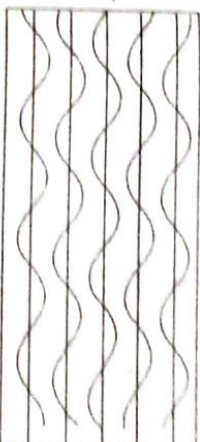
(b) The distance between a monochromatic source of light and a double slit is 2 meter. The wavelength of light is 6×10^{-7} m. The separation of S_1 & S_2 is 2 mm. What would be the size of the source of light so that spatial coherence is maintained between S_1 & S_2 ? (J.T.E. 2004)

(c) Show that the numerical aperture of a step index fiber is given by $N.A. = n_1 \sqrt{2\Delta}$, where symbol have their usual meaning? (J.T.E. 2004)

Sol.(a) Coherence: Two waves are said to be coherent if they have the same frequency (or wavelength) and are in phase (or have a constant phase difference between them). The phase means constant say over crests and troughs say over troughs. Light produced by lasers is coherent light. Light from light bulbs or the sun, however, is incoherent light.

Coherence is the co-ordinated motion of several waves maintaining a fixed and predictable phase difference.

Coherence is a concept that establishes the limits within which a real light source can be considered ideal. It is a measure of the correlation that exists between the phases of the wave measured at different points. The coherence of a wave depends on the characteristics of its source.



At any time, if the first wave changes, the second will change in the same way. If combined they can exhibit complete constructive interference at all times. It follows that they are perfectly coherent. Another necessary condition (for waves to be coherent) is therefore that both waves travel at the same speed.

The concept of coherence can be well understood from the Fig.(1) and (2). Fig.(1) shows a typical beam of light waves from an ordinary source travelling through space. One can see that these waves do not have any fixed relationship with each other. This light is said to be 'incoherent', meaning that the light beam has no internal order.

On the other hand, Fig.(2) shows the light waves within a highly collimated laser beam. All of these individual waves are in step, or 'in phase', with one another at every point. 'Coherence' is the term used to describe such a property of laser light.

Another way of saying the same thing is that coherence is a measure of the ability of a light source to produce high contrast interference fringes when the light is interfered with itself in an interferometer. High coherence means high fringe visibility with excellent contrast (i.e. good black and white fringes or black and whatever color the light is) and low coherence means washed out fringes and zero coherence means no fringes.

Temporal Coherence: Temporal coherence is a measure of the correlation between the phases of a light wave at different points along the direction of propagation. It is the correlation between the waves at one place at different times along the path of a beam is called 'temporal coherence'. It tells us how monochromatic a source is. It is related to the emitted line width.

Let us consider a single point on the wave front. There will be a phase difference between time, $t = 0$ and $t = \Delta t$ of the wave. If this phase difference remains same for any value of Δt , then the wave has perfect temporal coherence. But if this is only for a specific value of Δt , then the wave has partial temporal coherence.

In other words if the phase difference ($\Delta\phi$), measured at single point on a wave in the space at the beginning (t_1) and end (t_2) of a fixed time interval (Δt), does not change with time (t) or the phase difference measured at a time between any two fixed points x_1 and x_2 , spaced any distance apart along any ray, does not change with respect to distance, then the waves are said to possess temporal coherence.

Spatial Coherence: Spatial coherence is a measure of the correlation between the phases of a light wave at different points transverse to the direction of propagation. Spatial coherence tells us how uniform the phase of the wave front is. It is the correlation between different phases (but not along the path). In case, the phase difference remains same for any two points anywhere on the wave front, then the wave has perfect spatial coherence, whereas if this is true only for a specific area, then the wave is said to have only partial spatial coherence. Spatial coherence is related to directionality and uniphase wave fronts.

Sol.(b) The distance between a monochromatic source of light and a double slit (d) = 2 meter.

$$\text{Separation of } S_1, S_2, (y) = 2 \text{ mm} = 2 \times 10^{-3} \text{ meter}$$

$$\text{Wavelength of light } (\lambda) = 6 \times 10^{-7} \text{ meter}$$

$$\text{Let the size of source} = a$$

then spatial coherence will be maintained between S_1 & S_2 if

$$\frac{y}{d} \leq \frac{\lambda}{a}$$

$$\Rightarrow a \leq \frac{\lambda d}{y}$$

$$\Rightarrow a \leq \frac{6 \times 10^{-7} \times 2}{2 \times 10^{-3}}$$

$$\Rightarrow a \leq 6 \times 10^{-4} \text{ meter}$$

So the size of source must be less than or equal to 6×10^{-4} meter for spatial coherence.

Prob. 7 What is coherence? Explain temporal and spatial coherence. Show that visibility is a measure of a degree of coherence. [R.T.U. 2015]

Sol. Coherence : Refer to Long Answer Prob. 1(a).

Visibility as a Measure of Coherence

Let us consider two interfering waves, each of intensity I_0 . Both the waves consists of coherent part I_c and incoherent part I_{inc} . If C is the degree of coherence, then

$$I_c = C I_0 \quad \dots (1)$$

$$I_{inc} = (1 - C) I_0 \quad \dots (2)$$

When these waves superimposed, the coherent part will interfere by adding their amplitudes whereas for the incoherent parts the intensities are simply added. For constructive interference

$$I_{max} = 4 I_c + 2 I_{inc} \quad \dots (3)$$

Since for coherent part, amplitudes are directly added the term is $4 I_c$ [as $a_1 + a_2 = 2a$, and $(2a)^2 = 4a^2 = 4 I_c$]

where a_i is the amplitude corresponding to coherent part. For incoherent part, intensities are directly added i.e.

$$I_{inc} + I_{inc} = 2 I_{inc} \quad \dots (4)$$

Hence the resultant intensity for constructive interference is the sum of $4 I_c$ and $2 I_{inc}$

$$\text{Thus, we get } I_{max} = 4 C I_0 + 2 (1 - C) I_0 \quad \dots (5)$$

For destructive interference

$$I_{min} = 0 + 2 I_{inc} \quad \dots (6)$$

For coherent part amplitude in opposite phase will add to zero [as $a_1 + (-a_2) = 0$]

For incoherent part, intensities are directly added

$$I_{inc} + I_{inc} = 2 I_{inc} \quad \dots (7)$$

Thus $I_{min} = 0 + 2 (1 - C) I_0$

Now fringe visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad \dots (8)$$

$$V = \frac{4 I_c + 2 I_{inc} - 2 I_{inc}}{4 I_c + 2 I_{inc} + 2 I_{inc}}$$

$$V = \frac{4 C I_0 + 2 (1 - C) I_0 - 2 (1 - C) I_0}{4 C I_0 + 2 (1 - C) I_0 + 2 (1 - C) I_0}$$

$$V = \frac{4 C I_0}{4 I_0} = C \quad \dots (9)$$

Therefore, the fringe visibility V is the degree of contrast is a measure of degree of coherence between waves of equal intensities.

Prob. 8 Calculate the refractive indices of core and cladding materials of an optical fiber if its numerical aperture is 0.22 and relative refractive index difference is 0.012. [R.T.U. 2015, 13, 09]

Sol. Given : $NA = 0.22$

and $\Delta = 0.012$

$$NA = \mu_{core} \sqrt{2\Delta}$$

$$\mu_{core} = \frac{NA}{\sqrt{2\Delta}}$$

$$\mu_{core} = \frac{0.22}{\sqrt{2 \times 0.012}} = 1.42$$

$$\Delta = \frac{\mu_{core} - \mu_{clad}}{\mu_{core}}$$

$$\Rightarrow 0.012 = \frac{1.42 - \mu_{clad}}{1.42}$$

$$\Rightarrow \mu_{clad} = 1.42 - 0.012 \times 1.42 = 1.40$$

Prob. 9 Write two prominent applications of optical fiber. [R.T.U. 2015, R.T.U. (I Sem) 2008]

Sol. Applications of Optical Fiber

- Optical fibers are extensively used as information transmitting media due to its low loss over wide range of frequencies.
- Optical fibers find their applications in medical sciences as the image transmission devices as in flexible fibroscope.
- It is used in communication since on using conventional wire system due to various attenuations, the signal amplitude decreases but not in optical fiber.

- Typical bio-sensor designed using above fiber optics, may be used in measure of blood flow rate, dye dilution and pH etc.

Advantages of Optical Fiber

- They need no electric connections.
- They have high degree of mechanical flexibility.
- Low weight and low cost.

Prob. 10 Describe the construction of an optical fiber. What do you mean by numerical aperture of an optical fiber? Find an expression for the numerical aperture of an optical fiber. [R.T.U. 2015]

Sol. Construction : The optical fiber mainly consists of six layers as shown in Fig. 1.

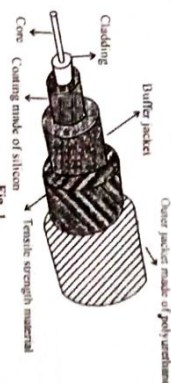


Fig. 1

Layers	Material	Size	Use
1. Core	Typical glass fiber	50 μm	Transmission of information
2. Cladding	Glass of slightly lower refractive index	125-200 μm	To improve the quality of transmission.
3. Silicon coating	Silicon liquid	-	Protect from moisture and absorption.
4. Buffer jacket	Plastic	-	To provide toughness and tensile strength.
5. Strength material	Kevlar	-	To provide toughness and tensile strength.
6. Outer jacket	Black polyurethane	1 mm	To protect bending, stretching and pulling.

Propagation of Light in an Optical Fiber : A light ray that enters the core of the fiber does not simply travel straight down through the centre of the core. Instead light rays within the core are continually reflected at the core-cladding boundary so that the rays remain within the core.

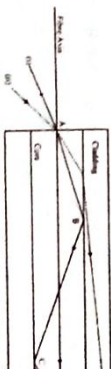


Fig. 2 : Propagation of Light in an Optical Fiber

This process is called total internal reflection and is the means by which an optical signal is confined to the core of fiber. Light pulses move easily down the fiber-optic line.

because of a principle known as **total internal reflection**. This principle of total internal reflection states that when the angle of incidence exceeds a critical value, light cannot get out of the glass, instead, the light bounces back in. When this principle is applied to the construction of the fiber-optic strand, it is possible to transmit information down fiber lines in the form of light pulses. Fig. 2 illustrates the process for a step index fiber.

In order to understand the process in more detail consider a light ray (1) entering the core at point A and then traveling through the core until it reaches the core-cladding boundary at point B. As long as the light ray intersects the core-cladding boundary at a small enough angle the ray will be reflected back into the core to travel on to point C where the process of reflection is repeated as shown in Fig. 2.

If a ray enters the fiber at a steep angle, for example light ray (ii), then when this ray intersects the core-cladding boundary, the angle of intersection is too large and reflection back in to the core does not take place and the light ray is lost in the cladding. This means that to be guided through a fiber a light ray must enter the core with an angle that is less than the intersection angle so called **acceptance angle** for the fiber. A ray, which enters the fiber, with an angle greater than the acceptance angle, will be lost in the cladding. By convention the 'acceptance angle' for a fiber can also be described by the term 'numerical aperture'. The fiber acceptance angle can be calculated from the refractive indices of the core and cladding using the formula:

$$\theta_A = \sin^{-1} \left(\sqrt{\mu_{core}^2 - \mu_{clad}^2} \right)$$

The numerical aperture of a fiber is simply equal to the mathematical sine of the fiber acceptance angle hence the numerical aperture (NA) is given by

$$NA = \left(\sqrt{\mu_{core}^2 - \mu_{clad}^2} \right)$$

Expression for NA : Refer to Long Answer Prob. 1(c).

Prob. 11 Calculate the temporal coherence length for

- Mercury vapour lamp emitting in green portion of spectrum at wavelength of 546.1 nm with emission bandwidth of $\Delta\lambda = 6 \times 10^6$ Hz.
- A helium neon laser operating at wavelength of 632.8 nm.

[R.T.U. 2014, 10]
[R.T.U. 2014]

Another aspect of coherence is that the interfering waves must have the same plane of polarization. If one has vibrations in a vertical YZ plane while travelling along X-axis and if the other has vibrations in a horizontal XY plane while travelling along the same X-axis, then the two crests shall not reinforce because they will be at right angles to each other. So, we conclude that interference will not be possible if the two light waves are differently polarized. From here, it follows that in the case of light, two separate independent sources can never be coherent. Thus, we must try to obtain a pair of coherent source from a single source of light as we do in Young's double slit experiment. In the case of Lloyd's mirror, the source along with its virtual image will act as a pair of coherent sources. In Fresnel's biprism, two virtual images of the same source will act as coherent sources.

Prob.20 For a step index fibre having core index 1.43 and cladding index 1.4, calculate :

(i) Critical angle

(ii) Critical propagation angle

(iii) Numerical aperture

[R.T.U. 2012]

$$\text{Sol. (i) } \sin \alpha_c = \frac{n_2}{n_1} = \frac{1.4}{1.43} = 0.979$$

$$\text{Critical angle, } \alpha_c = 78.24^\circ$$

$$\text{(ii) } \cos \theta_c = \frac{n_2}{n_1} = \frac{1.4}{1.43} = 0.979$$

$$\text{Critical propagation angle, } \theta_c = 11.76^\circ$$

$$\begin{aligned} \text{(iii) Numerical aperture} &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{1.43^2 - 1.4^2} = 0.29 \end{aligned}$$

Prob.21 Compare the maximum angle of acceptance and Numerical aperture of two fibres. Characterized by core and cladding index n_1 and n_2 respectively

(a) $n_1 = 1.6, n_2 = 1.5$

(b) $n_1 = 2.1, n_2 = 1.5$

[R.T.U. 2011]

Sol. The numerical aperture is given by

$$NA = \sqrt{\frac{\mu_{\text{core}}^2 - \mu_{\text{clad}}^2}{\mu_0}}$$

When the fiber is in air, $\mu_0 = 1$

$$\text{(a) } NA \text{ for I fibre} = \sqrt{(1.6)^2 - (1.5)^2} = 0.55$$

$$\begin{aligned} \text{(b) } NA \text{ for II fibre} &= \sqrt{(2.1)^2 - (1.5)^2} \\ &= 1.47 \end{aligned}$$

Maximum angle of acceptance is given by

$$\theta_{\max} = \sin^{-1} \sqrt{\frac{\mu_{\text{core}}^2 - \mu_{\text{clad}}^2}{\mu_0}}$$

$$\theta_{\max} = \sin^{-1} (NA)$$

$$\theta_{\max} \text{ for I fiber} = \sin^{-1} (0.55) = 33.36^\circ$$

$$\begin{aligned} \theta_{\max} \text{ for II fiber} &= \sin^{-1} (1.47) \\ &= 90^\circ - 53.57^\circ \end{aligned}$$

Prob.22 Explain the use of optical fibres in :

(a) Fibroscope

(b) Sensor

(c) Optical gyroscope

[R.T.U. 2011]

Sol. (a) Fibroscope : A fibroscope is a flexible fiber optic bundle with an eyepiece at one end, and a lens at the other. It is used for inspection work, often to examine small components in tightly packed equipment, when the inspector cannot easily access the part requiring inspection. The lens is often a wide-angle lens, and the eyepiece is occasionally instead connected to a camera.

Some fiberscopes use an additional fiber to carry light from an external source to illuminate the material being inspected, for clearer viewing. All fibroscopes introduce a certain amount of image distortion; much of this is similar to the distortion of modern night vision equipment.

Fiberscopes are used in medicine, machining, computer repair, and computer forensics, among many other uses.

(b) A fiber optic sensor is a sensor that uses optical fiber either as the sensing element ("intrinsic sensors"), or as a means of relaying signals from a remote sensor to the electronics that process the signals ("extrinsic sensors"). Fibers have many uses in remote sensing. Depending on the application, fiber may be used because of its small size, or

❑ **Laser** : The word LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. LASER is a device, which produces a highly intense, highly coherent parallel beam of monochromatic and coherent.

Laser plays an important role because of their inherent properties of high intensity, high degree of spatial and temporal coherence and polarization, find extensive application not only in various branch of engineering and technology but also in medical application, industry, entertainment, surgery, weapons, holography etc.

❑ **Spontaneous Emission and Stimulated Emission** : The atom or molecule remains in excited state, nearly about 10^{-8} sec. After this time, it will emits radiation again and return to a lower energy state. In this emission process the atoms spontaneously goes to a lower energy state through the emission of photon is called spontaneous emission or Fluorescence.

It is also possible that the emission is stimulated by incoming photons, which is called stimulated emission.

❑ **Essential Conditions for Laser Action**

- (a) Large number of photon of desired frequency and phase should be available to induce emission i.e. energy density of incident photon must be large.
- (b) The number of atoms in excited state must be larger than the number of atoms in ground state.
- (c) The probability of stimulated emission must be larger than the probability of spontaneous emission.

❑ **Pumping** : This is the process to achieve population inversion.

❑ **Population Inversion** : The situation in which the population density in the metastable state must be made higher than that of ground state.

❑ **Optical Resonator** : The system which provides positive feedback is called optical resonator energy. It consists of two reflecting surfaces.

❑ **Metastable State** : The state in which the atoms stay for a longer period (about 10^{-3} sec.) is known as metastable state.

❑ **Comparison between Ruby Laser and He-Ne Gas Laser**

	Ruby Laser	He-Ne Laser
1.	It is three energy level system.	It is four energy level system.
2.	Laser light from this laser is less collimated and coherent.	It gives highly coherent and collimated laser light.
3.	It produces pulsating laser beam.	It produces continuous laser beam.
4.	Cooling arrangement is required.	Cooling arrangement is not required.
5.	It emits light of wavelength 6943 Å.	It emits light of wavelength 6328 Å.

PREVIOUS YEARS QUESTIONS

I. SHORT ANSWER TYPE QUESTIONS

Prob.1 Give the reason for high intensity of a laser.

[R.T.U. 2018]

Sol. Due to its lower spreading property many beams of light incident in a small area, hence intensity of laser light is very sharp (intense). Due to this property it is used for the laser welding.

Prob. 2 What are the differences between spontaneous and stimulated emission?

(R.T.L. 2017)

Sol. Spontaneous Emission and Stimulated Emission :

The atom or molecule remains in excited state, nearly about 10^{-8} sec. After this time, it will emit radiation again and return to a lower energy state. In this emission process the atoms spontaneously goes to a lower energy state through the emission of photon is called spontaneous emission or fluorescence.

It is also possible that the emission is stimulated by incoming photons, which is called stimulated emission.

Prob. 3 Find the intensity of a laser beam of 20 mW power and having a diameter of 1.5 mm. Assume the intensity to be uniform across the beam.

(R.T.L. 2010)

Sol. Given: Power of Laser Beam $P = 20 \text{ mW} = 20 \times 10^{-3} \text{ watt}$
diameter $d = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

$$r = \frac{1.5 \times 10^{-3}}{2} \text{ m}$$

$$I = \frac{P}{A} = \frac{20 \times 10^{-3} \times 4}{\pi r^2} = \frac{3.14 \times (1.5 \times 10^{-3})^2}{2}$$

$$I = 1.13 \text{ kW/m}^2$$

Prob. 4 What do you understand by laser?

Sol. Laser : Lasers are amazing devices which emit beam of light powerful enough to vaporize a bulldozer, yet are so precise that they can be used in optical surgery, provide the surgeon removers to change the setting on the laser to 'delicate optical surgery' from vaporizes bulldozer.

Prob. 5 A laser beam can be focused on an area equal to the square of its wavelength λ , for a He-Ne laser $\lambda = 6328 \text{ \AA}$. If the laser radiates energy at the rate of 1 mW, find out the intensity of focused beam.

Sol. Given:

$$\text{Wavelength } \lambda = 6328 \text{ \AA} = 6328 \times 10^{-10} \text{ m}$$

$$\text{Power } P = 1 \text{ mW} = 10^{-3} \text{ W}$$

$$\text{Area } A = \lambda^2 = (6328 \times 10^{-10})^2 \text{ m}^2 = 40 \times 10^{-14} \text{ m}^2$$

$$\text{Intensity is given by } I = \frac{P}{A}$$

$$I = \frac{10^{-3}}{40 \times 10^{-14}} \text{ W/m}^2 = 2.5 \times 10^9 \text{ W/m}^2$$

Prob. 6 Give two important terms in interaction of light with matter.

Sol. There are two important terms in interaction of light with matter :

- Energy density of radiation :** It is defined as radiation energy per unit volume with in frequency range ν and $\nu + d\nu$.
- Population :** It is defined as the number of atoms per unit volume available in an energy level.

Prob. 7 Write down the components of a basic laser system.

Sol. It consists mainly of three basic parts :

- An active medium with a suitable set of energy levels to support laser action.
- A source of pumping energy into the device in order to establish a population inversion.
- An optical cavity or mirror arrangement that reflects the beam of light back and forth through the gain medium for further amplification and overcoming all losses.

Prob. 8 What do you understand by Helium-Neon laser?

Sol. A Helium-neon laser, usually called a He-Ne laser, is a small gas laser generally used in laboratory demonstrations of optics. Its usual operation wavelength is 632.8 nm or 6328 Å in the red portion of the visible spectrum. It produces a continuous laser beam which is highly directional, monochromatic and coherent.

Prob. 9 Give the advantages of Helium-Neon laser.

Sol. Advantages of Helium-Neon laser :

- It is most common and inexpensive gas laser.
- Typical He-Ne lasers can generate a few milliwatts of output power ranging from 1 mW to 100 mW in continuous-wave operation.
- The beam quality is usually excellent. It is often used for alignment purpose.

II. LONG ANSWER TYPE QUESTIONS

Prob. 1 Describe the construction and working of He-Ne laser. What is the role of He? Give some important applications of this laser.

(R.T.L. 2018)

Sol. Construction of He-Ne Laser : He-Ne laser consists of a narrow discharge tube of bore glass or quartz of length 10 to 100 cm with a diameter of order of 5mm to 10mm. The tube is filled with a mixture of helium and neon gases, approximately in the ratio 85% helium and 15% neon, contained at low pressure (typically $\sim 2-3$ torr). At each end of the tube there are electrical connections, referred to as electrodes.

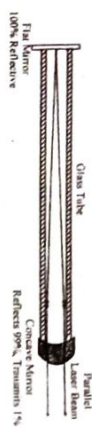


Fig.(a)

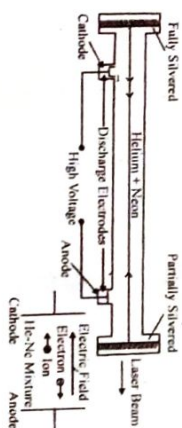
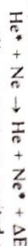


Fig.(b) : Schematic of a Helium-Neon Laser

The energy or pump source of the laser is provided by an electrical discharge of around 1000 V through an anode and cathode at each end of the glass tube. The positive high voltage is connected to the anode and cathode, is taken at ground potential. The anode consists of a metal pin inserted through the glass so that one end extends into the gas region of the tube and the other is connected to the power supply as shown in fig.(b). The laser tube is enclosed between a plane, high-reflecting mirror at one end and a concave output coupler mirror of approximately 1% transmission at the other end as shown in fig.(a). These mirrors could be flat, but this requires great precision in alignment, so the common laboratory He-Ne lasers are manufactured with the semiconfocal mirror arrangement. The spacing between the mirrors is taken equal to an integral multiple of half wavelength of laser light.

Working : The laser process in a He-Ne laser starts with collision of electrons with the helium atoms in the gas

by the electrical discharge. This electric discharge excites helium from the ground state, to the long-lived, metastable excited states. Collision of the excited helium atoms with the ground-state neon atoms results in transfer of energy to the neon atoms, exciting them into the 2s and 3s states. The excess energy in a helium atom is transferred during a collision to a neon atom. Helium is used because, it possesses an excited state very close to upper (3s) level in neon and this helium level is metastable, meaning it has a very long lifetime (as one of the excited levels of helium at 20.61 eV is very close to a level in neon at 20.66 eV, so close in fact that upon collision of a helium and a neon atom, the energy can be transferred from the helium to the neon atom). The helium excited state is readily produced by electrical discharge and such a large population of excited helium atoms can be created that a population inversion of neon atoms can be made. This process is given by the reaction equation :



where (*) represents an excited state, and ΔE is the small energy difference between the energy states of the two atoms, of the order of 0.05 eV.

To ensure the emission processes exceed the absorption process, there must be a population inversion, namely an excess of excited states. The number of neon atoms entering the excited states increases as further collisions between helium and neon atoms occurs, causing a population inversion between the neon 3s and 2s, and 3p and 2p states.

Role of He in Laser : The role of helium gas in the laser tube is to provide the pumping medium to neon atom to attain the necessary population inversion for laser action.

Applications of He-Ne Laser

The fields in which helium-neon laser is applied are appended below:

- Interferometry
- Laser printing
- Bar-code reading
- As pointing and directional reference beams.

In interferometry the helium-neon laser gives a very stable and single transverse mode reference beam, which is essential in identifying optical characteristics of material like smoothness (λ) and the figure of the surface.

In laser printing the laser beam is used as a source for writing on the photo-sensitive material, which give print patterns in detail.

It is now common to see departments stores using He-Ne lasers to check out or for the scanning the merchandise having digital codes, which are imprinted on the product. The billing and payment takes only seconds.

Prob.2 What is Laser? Explain its principle?

[R.T.U. 2017]

Sol. Laser : Lasers are amazing devices which emit beam of light powerful enough to vaporize a bulldozer, yet are so precise that they can be used in optical surgery. provide the surgeon remembers to change the setting on the laser to 'delicate optical surgery' from vaporizes bulldozer

Principle of Laser : The principle of laser production is based on the fact that atoms of a material have a number of energy levels in which at least one is metastable state

Consider a three level atomic system having energies E_1 , E_2 and E_3 respectively. Let the atoms are at ground state E_1 . If photons interact with an atom in ground state, the atom absorbs the photon and reaches the excited state E_3 . We know that the excited state is an unstable state, therefore, electron must return back to ground state E_1 but such transitions are not allowed and the electron first reach the state E_2 . Atoms in the state E_2 which has a life time of about 10^{-8} sec decay spontaneously from state E_2 to state E_1 which is metastable and has life time of 10^{-3} sec. This means that the atoms reach state E_2 much faster than they leave state E_2 . This results in an increase in number of atoms in state E_2 and hence population inversion is achieved.

After achieving population inversion it is exposed to a beam of photons which causes induced emission of photons and a beam of laser is produced.

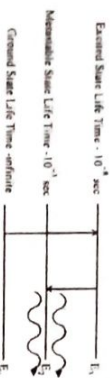


Fig. : Laser Principle

Prob.3 What are basic requirements of semiconductor laser? Draw its label diagram and explain its construction working with necessary theory. Write down the applications of laser. [R.T.U. 2016, 2014, 10]

OR

What are the basic requirements of a semiconductor laser? Draw its labelled diagram and explain its working with necessary theory. Write down the applications of semiconductor laser. [R.T.U. 2017]

OR

Describe the principle, construction and working of a semiconductor laser. Describe various applications of semiconductor laser. [R.T.U. 2013]

OR

Write short note on semiconductor laser. [R.T.U. 2011]

Sol. Principle of Laser : Refer to Prob.2

Essential Requirements for Producing Laser Action :

1. The energy density of incident radiation must be very large. This can be achieved by reflecting the incident radiation back and forth in the active medium by putting two mirrors in proper orientation. This system provides positive feedback is called optical resonator.
2. The rate of stimulated emission must be very large as compared to spontaneous emission. To increase the rate of stimulated emission, metastable states of higher energy is chosen because transitions from metastable state by spontaneous emission are not allowed.
3. Total emission rate must be greater than absorption rate. Thus, to increase emission rate we have to increase number of atoms in higher energy state. The situation in which the number of atoms in higher energy state exceeds that in lower energy state is known as population inversion.

Population inversion can be achieved by pumping mechanism. **Construction : Basic Semiconductor Laser Design**

A schematic diagram of the simplest type of gallium arsenide laser is shown in Fig.1(a, b). It is a solid-state device that consists of two outer semiconductor layers separated by a middle layer and generates laser radiation, when charge carriers of opposite polarity, one each from the top and bottom layers, meet in the middle-layer. Such lasers consist of two basic components, an optical amplifier and a resonator. The amplifier is made from a direct-band gap semiconductor material based on either gallium arsenide (GaAs) substrates, which is a small, highly efficient photon source. GaAs cleaves easily along certain crystal planes, leaving flat parallel surfaces. The cleaved ends of the laser diode, with no further coating form the mirrors for feedback and output coupling. Excitation is provided by current flow through the device as the top and bottom layers are metallized and metal contacts are provided. If output is desired only from one end of the device it can be achieved by coating the surfaces with metal films. Two sides are purposely roughened to reduce reflection and prevent lasing 'across' the diode cavity.

Working : The radiative recombination of electron-hole pairs can be used for the generation of electromagnetic radiation by the electric current in a p-n junction. This effect is called electroluminescence. When a diode is forward biased, holes from the p-region are injected into the n-region, and electrons from the n-region are injected into the p-region,

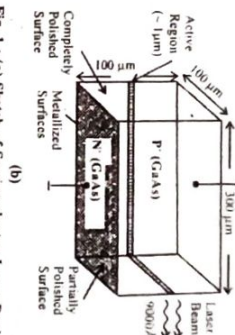
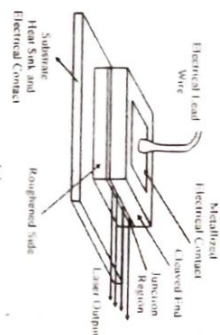


Fig.1 : (a) Sketch of Semiconductor Laser Device
(b) Construction of a Semiconductor Diode Laser

If electrons and holes are present in the same region, they may radiatively recombine that is, the electron falls into the hole and emits a photon with the energy of the bandgap. This is called spontaneous emission, and is the main source of light in a light emitting diode. Under suitable conditions, the electron and the hole may coexist in the same area for quite some time (on the order of microseconds) before they recombine. If a photon of exactly the right frequency happens along with this time period, recombination may be stimulated by the photon. This causes another photon of the same frequency to be emitted, with exactly the same direction, polarization and phase as the first photon. As the mirrors are provided by cleaved crystallographic surfaces of the semiconductor and the concentration of the electron hole pairs (called the injection level) exceeds some critical value, this device may function as a semiconductor laser that emits a coherent electromagnetic radiation with all photons in phase with each other.

In a laser diode, the semiconductor crystal is fashioned into a shape somewhat like a piece of A4 paper-very thin in one direction and rectangular in the other two. The top of the crystal is n-doped, and the bottom is p-doped, resulting in a large, flat p-n junction. The two ends of the crystal are cleaved so as to form perfectly smooth, parallel edges, two reflective parallel edges are called a Fabry-Pérot cavity. Photons emitted in precisely the right direction will be reflected several times

from each end face before they are emitted. Each time they pass through the cavity, the light is amplified by stimulated emission. Hence, if there is more amplification than loss, the diode begins to 'lase'. This type of laser diode is called homojunction laser diode.

Emission of Light by Semiconductor Diodes

Fig.2 shows the relative populations of the energy bands at both sides of a p-n junction with no voltage applied to the diode

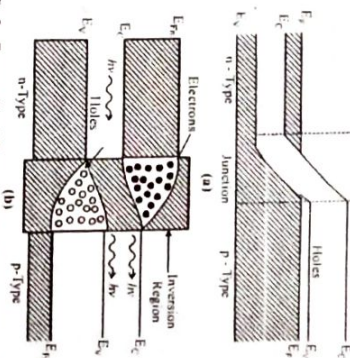


Fig.2 : Energy Band Structure of a Semiconductor Diode

(a) Heavily Doped PN-junction Forward Biased above Threshold Value

(b) Heavily Doped PN-junction Forward Biased below Threshold Value

The n-type material contains electrons, which behave as the current carriers in its conduction band, whereas the p-type material has holes for carriers in its valence band. Because of very high doping on n-side the donor levels are broadened and extended into the conduction band. Fermi level is also pushed into the conduction band hence electron occupy the portion of conduction band lying below the Fermi level. Similarly on the heavily doped p-side Fermi level lies within the valence band and holes occupy the portion of valence band that lies above the Fermi level. At equilibrium, the Fermi level is uniform across the junction.

When a forward voltage is applied to the diode, the energy levels are caused to shift as illustrated in Fig.3. Under these conditions there is a significant increase in the concentration of electrons in the conduction band near the junction on the n-side and the concentration of holes in the valence band near the junction on the p-side. The electrons and holes recombine and energy is given off in the form of photons. The energy of the photon resulting from this recombination is equal to that associated with the energy gap. In light-emitting diodes (LED) this light energy is transmitted out through the sides of the junction region. For the LED, all

of the light is created by spontaneous emission due to electron and hole recombination. In semiconductor lasers the junction forms the active medium, and the reflective ends of the laser material provide feedback. Because of this feedback in diode lasers, most of the light is created by stimulated emission.

Applications of Laser

Laser plays an important role due to its outstanding characteristics in different areas of research and development of science, engineering and technology. Generally speaking, a laser produces a very narrow and intense beam of light that can vaporize anything which comes in its way. The lasers have widespread applications in industry, surgery, entertainment etc. and in a number of other disciplines of life. However, some of them are as under:

- (i) **Industry** : In industries precision cutting, welding and drilling of tiny holes into hardest materials can be performed by laser beam.
- (ii) **Electrical Devices** : Tiny solid-state lasers are widely used in electrical devices. Lasers are a primary component of some of our most modern communication systems and are the probes that generate the audio signals from our compact disk players. A laser beam replaces the photographic needle in a compact disk audio system for music reproduction of extremely high fidelity. A laser can also be used for the photographic recording of output data of computer.
- (iii) **Media** : All T.V. transmission of programs to the entire world is an application of laser. In fibre optic communication laser plays a vital role.
- (iv) **Surgery** : In surgery, the laser has proved a more delicate and accurate instrument than the finest scalpel. It has been used for bloodless surgery, removing small tumors, cutting and delicate operations. Laser can also be used to fragment gallstones, stones in kidney and eye-surgery. Laser therapy is completely painless.
- (v) **Military** : Lasers are the targeting elements of laser-guided bombs. Laser beam is used as a range finder, lining up equipment accurately in surveying large distances.
- (vi) **Market** : Lasers are the optical source in both supermarket checkout scanners and tools (steppers) that print our microchips.

(vii) **Environmental Study** : Laser is an important environmental monitoring instrument. Some types of laser can be tuned to a desired frequency. These lasers can be used as very sensitive detectors of pollutants in the atmosphere.

(viii) **Nuclear Fusion** : A high power laser is also a potential source to induce nuclear fusion reaction.

(ix) **Recording Hologram** : A very interesting application is the production of true three-dimensional images called **Hologram**. A hologram records not only the intensity of the light but also the phase difference between two image-beams that cause the interference pattern to reconstruct an image suspended in air like a real object.

(x) **Education and Research** : Laser helps us to investigate the structure of molecules and compounds. It can be used as a catalyst in some chemical reaction. It can be used for alignment purpose in scientific and commercial instruments.

Prob-4 Explain Laser action and give the reasons for the following basic properties of a laser

- (i) **High intensity**
- (ii) **High directionality**

[R.T.U. 2016]

Sol. Laser Action : When energy is applied to a medium raising electrons to an unstable energy level. These atoms spontaneously decay to a relatively long-lived, lower energy, meta-stable state. A population inversion is achieved when the majority of atoms have reached this meta-stable state. Lasing action occurs when an electron spontaneously returns to its ground state and produces a photon. If the energy from this photon is of the precise wavelength, it will stimulate the production of another photon of the same wavelength and resulting in a cascading effect. The highly reflective mirror and partially reflective mirror continue the reaction by directing photons back through the medium along the long axis of the laser. The partially reflective mirror allows the transmission of a small amount of coherent radiation that we observe as the 'beam'. Laser radiation will continue as long as energy is applied to the lasing medium. Three – level laser production is given below :

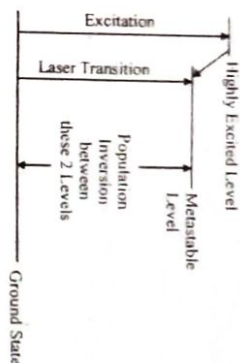


Fig. 1 : Three Level Laser Action

Reasons for the Basic Properties : Laser light source when compared with any conventional light source, it shows some extraordinary characteristics given as follows:

(i) **Monochromaticity** : The property of exhibiting the emission of a single wavelength by a light source is called monochromaticity. When laser is passed through a prism then a single line will be appeared in the optical spectrum, but not in case of ordinary light (Fig.).

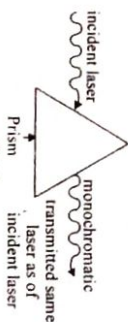


Fig. 2

(ii) **Directionality** : During the propagation of laser its angular spreading is very small and possessed very less where it is incident.

(iii) **Brightness or Intensity** : Due to its lower spreading property many beams of light incident in a small area, hence intensity of laser light is very sharp (intense). Due to this property it is used for the laser welding.

(iv) **Coherence** : In case of conventional light, the property of coherence exists between a source and its virtual source where as in case of laser the property of coherence exists between any two or more light wave of same type.

Prob-5 Explain the term absorption, spontaneous emission and derive the relation between Einstein's coefficients for laser action and discuss the results.

[R.T.U. 2016]

OR
Derive the relation between Einstein's coefficients and discuss the results.

[R.T.U. 2015]

OR
Explain the term absorption, spontaneous emission and stimulated emission and derive a relation between Einstein's Coefficients.

[R.T.U. 2014]

OR
What do you understand by spontaneous emission and stimulated emission? Obtain a relation for Einstein coefficients A and B for spontaneous and stimulated emission. What are essential requirements for Laser action?
[R.T.U. 2010, 2009, (II Sem) 2008]

OR
Explain the meaning of "stimulated emission".
[R.T.U. 2007]

Sol. Absorption : To explain the absorption, we discuss an atom with two active energy levels E_1 and E_2 , where E_1 is ground state and E_2 is excited state. The atoms will stay in the ground state energy level till the external energy is not applied as shown in Fig. 1.

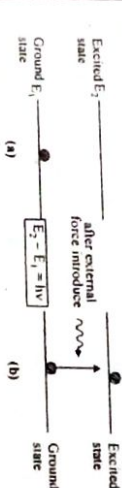


Fig. 1

When a photon of energy $h\nu = E_2 - E_1$ is incident on the atom lying in ground state then it excites to higher state E_2 . The difference of energy of these energy states is absorbed by the atom. This probability of transition is called the absorption.

The rate of probability of absorption phenomenon from energy state E_1 to E_2 depends on number of atoms (N_1) available, permit volume and number of incident photon density (ρ_ν) then

$$R_{\text{Abs}} \propto N_1 \rho_\nu$$

$$R_{\text{Abs}} \propto N_1 \rho_\nu = \beta_{12} \rho_\nu N_1$$

$$R_{\text{Abs}} = B_{12} \rho_\nu N_1 \quad \dots (1)$$

B_{12} → Coefficient of probability of transition from energy state E_1 to E_2 .

Spontaneous Emission : Let us consider that the atoms are in the excited state E_2 . After a particular time the atom de-excites to its ground state spontaneously by emitting a photon of energy $h\nu = E_2 - E_1$.

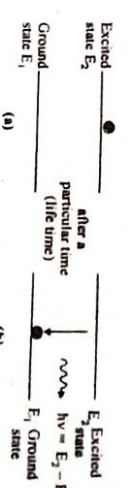


Fig. 2

However this process does not require any external energy i.e. this process is known as spontaneous (automatic) emission as shown in Fig. 2

The rate of spontaneous emission is proportional to the population (number of atom in the excited state only.)

$$R_{\text{STE}} \propto N_2 = A_{21} N_2 \quad \dots (1)$$

A_{21} - Coefficient of probability of spontaneous transition from energy E_2 to E_1

A_{21} is always zero i.e. $A_{21} = 0$, because there is not any possibility of transition without external energy from energy state E_2 to E_1 .

Stimulated Emission: Let us assume that the atom is in the excited state E_2 , if a photon of energy $h\nu = E_2 - E_1$ is incident on just before the lifetime, it gives an external energy to the atom, it stimulate these atoms E_2 to energy state E_1 . In result a photon of energy $h\nu$ releases along with the incident photon energy $h\nu$. This phenomenon is known as stimulated (induced) emission. The emitted photons in this process containing the same wavelength and they are in same phase as shown in Fig. 3

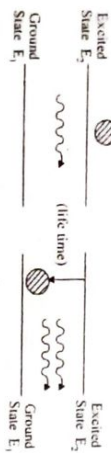


Fig. 3

The rate of transition probability of stimulated atom from energy state E_2 to energy state E_1 depends on the number of excited state E_2 and energy density of incident photons (ρ_v)

$$R_{\text{STE}} \propto N_2$$

$$R_{\text{STE}} \propto \rho_v$$

$$R_{\text{STE}} = B_{21} N_2 \rho_v$$

where B_{21} = coefficient of stimulated emission.

Theory of Laser: Einstein's Coefficient: In thermal equilibrium, the population of atoms in different states are governed by Maxwell-Boltzmann's Law. In the equilibrium condition,

Rate of transition of total emission = Rate of transition of total absorption.

$$R_{\text{AS}} = B_{12} N_1 \rho_v = \text{Absorption (induced)} \quad \dots (1)$$

$$R_{\text{STE}} = B_{21} N_2 \rho_v = \text{Stimulated emission} \quad \dots (2)$$

$$R_{\text{STE}} = A_{21} N_2 = \text{Spontaneous emission} \quad \dots (3)$$

$$A_{21} = 0 \text{ for automatic absorption} \quad \dots (4)$$

Prob. 6 Write short notes on the following:

- (i) Population inversion
(ii) Pumping

(R.T.U. 2015)

Sol. (i) Population Inversion: Emission of high intensity of radiation by stimulated emission in laser requires a large amount of excited atoms in the upper levels. Normally the higher energy levels have less occupancy than the lower levels. However, for laser emission we need to have the opposite. This population inversion is an artificial situation required to achieve stimulated radiation. The number of active atoms occupying an energy state is called population of that state.

We consider N_1 and N_2 are the populations of the lower E_1 and upper E_2 energy levels respectively. In thermal equilibrium these populations are fixed by the Boltzmann factor. From Boltzmann's statistics,

$$N_1 \propto e^{-E_1/kT} \text{ and } N_2 \propto e^{-E_2/kT}$$

$$\text{Hence, } \frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT} \quad \dots (1)$$

The negative sign in the exponent implies that $N_2 \ll N_1$ at equilibrium. Here more atoms are in lower energy level E_1 . This is the normal state. Atoms at E_1 will occasionally absorb radiation and make transition to E_2 . Atoms in E_2 will occasionally emit radiation and make transition to the lower level. This is spontaneous emission. Laser operation requires obtaining stimulated emission almost exclusively. There must be more atoms in the upper level than in the lower level in order to achieve stimulated emission exclusively. Therefore, a non-equilibrium state is to be produced in which the population of the upper energy level exceeds to a large extent the population of the lower energy level. When this situation occurs, the population distribution between the levels E_1 and E_2 is said to be inverted and the medium is said to have gone into the state of population inversion. Extending eq. (1) in this case, we observe that N_2 can exceed N_1 only if the temperature is negative. Therefore, the state of population inversion is referred to as a negative temperature state. The name only suggests that this state is a non-equilibrium state. The population inversion is actually achieved at normal temperature.

(ii) **Pumping:** The process of achieving population inversion is known as pumping.

Here the numbers of particles (N_2) in higher energy state E_2 becomes more. Sustaining the condition of population inversion requires the atoms have to be supplied continuously to excited state by any means.

This process of supplying energy to the medium to transfer it into the state of population inversion is known as pumping.

Prob. 7 Explain the construction and working of a He-Ne laser. Draw necessary diagram. What is the role of He in this laser?

(R.T.U. 2015, 2007)

OR

In He-Ne laser, what is the function of He atoms? Explain the answer with the help of energy level diagram for He-Ne. Describe with a neat sketch the working of He-Ne laser.

(R.T.U. 2013)

Sol. Refer to Long Answer Prob. 1.

Prob. 8 (a) Explain the principle of laser. What are the essential requirements of a laser? Explain how these requirements are achieved.

(b) Laser action occurs by stimulated emission from an excited state to a state of energy 30.5 eV. If the wavelength of laser light emitted is 690 nm, what is the energy of the excited one? (R.T.U. 2012)

Sol. (a) Principle of Laser: Refer to Long Answer Prob. 2.

Basic Requirement of Laser: Refer to Long Answer

Prob. 3

LASER action consists of following necessary conditions or parameters.

I. Active Medium

The active medium consists of a collection of atoms, molecules or ions (in solid, liquid or gaseous form) which is capable of amplifying light waves or medium in which light gets amplified is called active medium. A small fraction of active medium is responsible for amplification. These are called active centres and remaining part of this medium supports the active centres.

(i) This is the medium where light is getting amplified. This may be solid, liquid or gas.

(ii) This is the medium where only a fraction of atoms of a particular species takes part in stimulated emission in laser action. These atoms are also known as Active Centres.

II. Population Inversion: Refer to Long Answer Prob. 6(i).

(a) Factors affecting population inversion: There are three phenomena that can prevent population inversion. First one prevents lower laser level from undergoing its normal decay process in a gas laser. This is known as Radiation Trapping. The second, applicable to gas or plasma lasers, tends to reduce the population of the upper laser level and move it to the lower laser level via collisions. This is the Electron Collisional Thermalization. The third is the absorption within the gain medium by various processes present within the medium that are not associated with upper and lower laser levels.

(b) **Negative absorption** : Intensity of light decreases exponentially as it propagates through a medium as $I = I_0 e^{-\alpha L}$.

Here I_0 is the initial intensity; L is the length traveled by light in the medium and α is the absorption coefficient. So, the amplification of light may be described by the above equation but with negative α . In this point of view a medium in the state of population inversion is called a medium with a negative absorption coefficient and the light amplification is referred to as negative absorption.

Here the numbers of particles (N_2) in higher energy state E_2 becomes more than the same (N_1) in lower energy state E_1 , unlike normal situation. Thus the name 'Population Inversion' arises.

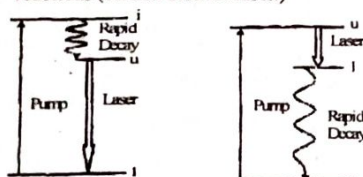
This is possible only when temperature is negative under normal case. So it is also referred as Negative Temperature State.

However, this is totally hypothetical as we can not go below 0K and it is obtained generally at room temperature. Generally light intensity decreases while propagating into a medium, so a medium in a state of population inversion is also called a medium of Negative Temperature Coefficient.

(c) **Pumping** : Refer to Long Answer Prob. 6(ii).

There are several methods of pumping processes :

- Optical pumping** using light source (in Solid State Laser, e.g. ruby).
- Electrical pumping** using gas discharge (in Gas Laser, e.g. He-Ne).
- Direct pumping** using electrical energy which is ultimately converted into light energy (semiconductor laser).
- Chemical pumping** using energy from chemical reactions (carbon dioxide laser).



(a) Traditional Solid State Laser
Fig. : Three Level Lasers

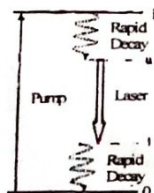


Fig. : Four Level Laser

Although it seems to have two energy to attain laser action, however, this is not possible. So, three or four level pumping schemes are often used and four level pumping shows maximum lasing efficiency. The base level is called the lower lasing level and the upper level as upper lasing level. The topmost level is known as pumping level. So, for the four level lasers we have two upper levels. In three level schemes laser transition takes place between upper and lower lasing level. However, the laser transition is in between two upper levels only in four level laser. The general energy level diagrams of three and four level laser are shown in Fig.

III. Optical Resonator

For the continuation of stimulated emission and the increase of light intensity in a cumulative way a positive feedback of light into the medium is a must.

Spontaneous photons are emitted by atoms independently at random which produce spontaneous emission in different directions. So the final result is incoherent light.

An intense beam of light can be available only when the optical gains of the system equals the optical losses.

All these three requirements are met with an Optical Cavity Resonator, which converts the active medium into a Light generator.

An optical resonator consists of two opposing plane parallel mirrors with the active material placed in between. One mirror is semi-transparent and the other one is totally reflecting. The mirrors are set normal to the optic axis of the materials. Now, when in an active medium pumping is taking place to achieve population inversion state. Spontaneous photons are emitted in initial stage. In order to have a coherent light output, we should have to select photons of specific directions and other are treated as reject one and these stimulated photons made to pass through the medium a number of times. So, here the mirrors constituting the resonator cause the directional selectivity. Photons emitted in any direction other than parallel to the optic axis, will pass out from the sides of the resonator and are lost. Photons emitted parallel to the axis are augmented. On reaching semi-transparent mirror some are transmitted out and rest reflected and they de-excite more atoms and build up their strength. At fully reflecting mirror major portion will be reflected. At each reflection at the front mirror, the beam is partially transmitted through it and partially fed back into medium. Thus, the photon density of the medium enhances. As the oscillations build up to enough intensity it emerges out through the front mirror as a highly collimated intense beam of laser.

Sol. (b) Energy of the laser light emitted is given as

$$h\nu = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m/s})}{(690 \times 10^{-9} \text{ m})}$$

$$= 2.88 \times 10^{-19} \text{ J}$$

$$= \frac{2.88 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 1.8 \text{ eV}$$

$$\text{Energy of the excited state} = 30.5 \text{ eV} + 1.8 \text{ eV}$$

$$= 32.3 \text{ eV}$$

Prob.9 The pulse width of laser of wavelength 1064 nm is 25ms. If the average power output per pulse is 0.8 W then

- What is the energy released per pulse?
- How many photons does each pulse contain?

[R.T.U. 2012]

Sol. (i) Energy released per pulse = $P \times \Delta t$

$$= (0.8 \text{ W})(25 \times 10^{-3} \text{ s})$$

$$= 20 \times 10^{-3} \text{ J}$$

(ii) Number of photons in a pulse semi-transparent

$$= \frac{\text{Energy}}{h\nu} = \frac{P \times \Delta t}{\frac{hc}{\lambda}}$$

$$= \frac{(20 \times 10^{-3} \text{ J})(1064 \times 10^{-9} \text{ m})}{(6.626 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m/s})}$$

$$= 1.07 \times 10^{17}$$

Prob.10 A laser beam has a power of 50 mW. It has an aperture of 5×10^{-3} and wavelength 7200 Å. A beam is focussed with a lens of focal length 0.1m. Calculate the area and the intensity of the image.

[R.T.U. 2012]

Sol. Given : Wavelength $\lambda = 7200 \text{ Å} = 7.2 \times 10^{-7} \text{ m}$

Aperture $a = 5 \times 10^{-3} \text{ m}$

Focal length of the lens = 0.1m

Power = $50 \times 10^{-3} \text{ watt}$

The angular spread $d\theta = \frac{\lambda}{a} = \frac{7.2 \times 10^{-7}}{5 \times 10^{-3}}$

$$= 1.44 \times 10^{-4} \text{ radian}$$

Areal spread = $(d\theta \cdot f)^2 = (1.44 \times 10^{-4} \times 0.1)^2$

$$= 2.074 \times 10^{-4} \text{ m}^2$$

PREVIOUS YEARS QUESTIONS

I. SHORT ANSWER TYPE QUESTIONS

Prob.1 What is Hall Effect? Write expression for Hall coefficient. [R.T.U. 2018]

Sol. Hall Effect : When a magnetic field is applied perpendicular to a current carrying conductor or semiconductor, a voltage is developed across the specimen in a direction perpendicular to both the current and the magnetic field. This phenomenon is called the Hall Effect and the voltage so developed is called Hall voltage.

Hall Coefficient : The average drift velocity $\bar{v}$ is related to the current density $\bar{j}$ as

$$\bar{j} = nq\bar{v}$$

$$v = \left| \frac{\bar{j}}{nq} \right|$$

$\bar{j}$ will be in same direction for both negative and positive charge carriers)

$$E_1 = vB$$

$$= \frac{1}{nq}(JB)$$

If the thickness of the bar is w , then

$$E_1 = \frac{V_H}{w}$$

Where V_H is potential difference between the edges.

V_H is called the Hall voltage and the factor $\left(\frac{1}{nq} \right)$ is called Hall coefficient and denoted by R_H .

Prob.2 Explain origin of bands in solids. [R.T.U. 2017]

Sol. According to the Bohr's theory, free electrons in an isolated atom that have certain definite discrete amount of energy. There are well maintained energy levels of electrons in each atom. If we bring two atoms near to each other then they both will affect each other. The interatomic interactions of atoms are very strong. So, it will not affect the electrons present in the inner shells but will affect the electrons in the outer shells. When atoms approach one another, it becomes possible for electron to exchange places. As a result of such transitions, identical energy levels split.

Prob.3 A silicon plate of thickness 1 mm, breadth 1 cm and length 10 cm is placed in a magnetic induction 0.5 wb/m² acting perpendicular to its thickness. If 1A current flows along its length, calculate the hall voltage developed if the hall coefficient is $3.66 \times 10^{-4} \text{ m}^3/\text{C}$.

Sol. Given : $B = 0.5 \text{ wb/m}^2$, $I = 1 \text{ Amp}$, $t = 1 \text{ mm}$,

$w = 1 \text{ cm}$, $l = 10 \text{ cm}$, $R_H = 3.66 \times 10^{-4} \text{ m}^3/\text{C}$

Hall voltage $V_H = \frac{R_H IB}{t}$

$$V_H = \frac{3.66 \times 10^{-4} \times 1 \times 0.5}{1 \times 10^{-3}}$$

$$= 0.183 \text{ V}$$

Prob.6 Explain the term bonding in crystals? What are the distinguishing characteristics of metallic bonding?

[R.T.U. Jan. 2016]

OR

Describe nature and origin of various forces existing between atoms of solid crystal and explain the formation of covalent and metallic bonding in solids.

[R.T.U. 2014]

OR

What do you mean by the term 'bonding in solids'. Explain all types of bonding.

[R.T.U. 2013]

Sol. Refer to Long Answer Prob.1(a).

Prob.7 Explain Hall effect. Obtain the expression for Hall coefficient. Hall voltage, Hall angle and Hall mobility.

[R.T.U. 2014]

OR

What is Hall effect? Briefly discuss the physical origin of Hall effect.

[R.T.U. 2016]

Sol. Hall Effect : Refer to Long Answer Prob.2(a).

Theory

Consider a conducting bar placed along the x-z plane in which an electric current I is flowing in the x-direction by the application of an electric field $\vec{E}$. A magnetic field $\vec{B}$ is applied in the z-direction as shown in Fig.1

Due to the electric field $\vec{E}$ in x-direction, the negative charge carriers (electrons) will be drifting in the (-x) direction while the positive charge carriers (ions or holes) will drift in (+x) direction. If the bar is a metal bar, the charge carriers will be electrons and if it is an extrinsic semiconductor the majority charge carriers will be electrons in N-type and holes in P-type.

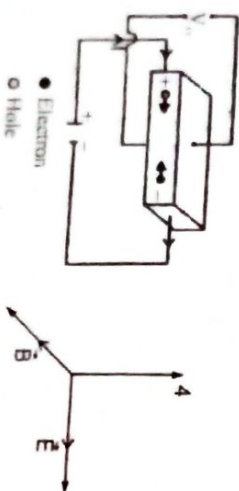


Fig. 1 : Hall effect

Let q be the charge on carriers and $\vec{v}$ be their average drift velocity. Due to the external magnetic field $\vec{B}$ a force will act on the moving charge carriers. This force will be

$$\vec{F} = q(\vec{v} \times \vec{B})$$

...(1)

For electrons $q = -e$, $\vec{v} = v(-\hat{x})$

and $\vec{B} = B(\hat{z})$

$$\vec{F} = -e[v(-\hat{x}) \times B(\hat{z})]$$

$$= evB(-\hat{y})$$

...(2)

i.e. the electrons will suffer a downward force shown in fig.(2)

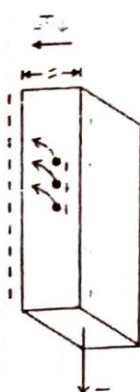


Fig. 2 : Downward force on electrons

For holes $q = +e$, $\vec{v} = v(\hat{x})$

and $\vec{B} = B(\hat{z})$

$$\vec{F} = e[v(\hat{x}) \times B(\hat{z})]$$

$$= evB(\hat{y})$$

...(3)

i.e. holes will also suffer a downward force.

With electrons as a charge carriers the electrons will pile up at the bottom of the bar and the top of the bar will become positively charged due to lack of electrons. As a result a transverse electric field $\vec{E}_t$ called Hall field will be produced which will oppose the downward movement of electrons. In the steady state which is attained in a very short time, the downward force due to magnetic field will be balanced by the upward force due to transverse electric field. The electric field $\vec{E}_t$ so developed will be in $(-\hat{y})$ direction. In the same way with positive charge carriers, they will also be deflected downward and will pile up on the bottom of the bar.

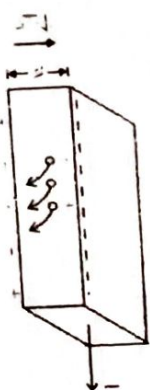


Fig. 3 : Downward force on holes

But now the transverse electric field $\vec{E}_t$ will be upward (+y-direction) opposing the downward movement of positive charge carriers. Again a steady state will be achieved when the force due to magnetic field will be balanced by the force

In steady state

$$q(\vec{v} \times \vec{B}) = -q\vec{E}_t$$

$$\vec{E}_t = -(\vec{v} \times \vec{B})$$

$$= -vB(-\hat{x} \times \hat{z})$$

$$= vB(-\hat{y}) \text{ (For electrons)}$$

...(4)

$$\text{and } \vec{E}_t = -(\vec{v} \times \vec{B})$$

$$= -vB(-\hat{x} \times \hat{z})$$

$$= vB(\hat{y})$$

...(5)

(For positive charge carriers)

Hall Coefficient : Refer to Short Answer Prob.1.

Prob.8 Explain what is 'covalent bonding'. Explain how the force of attraction between two atoms or ions as those are brought closer.

[R.T.U. 2015]

Sol. Covalent bond : Refer to Long Answer Prob.1(a).

Variation in Force of Attraction

There is a force of attraction F_c set up between oppositely charged ions which follows the well-known coulombic relationship $F_c = K/a^2$, where a is the separation distance between the two atoms.

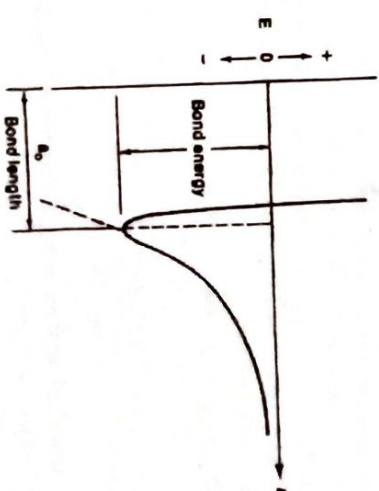


Fig. : General Shape of Bond Energy Curves for Covalent, Ionic, or Metallic Bonding

Note that once a charged ion is formed, it is independent of the other atom which took part in the electron transfer and can associate with or bond with any other ion of opposite charge. The proportionality K_c is a function of the valence Z .

Prob.11 (a) Define fermi function and fermi energy.

(b) Explain with graph, the variation of Fermi function with temperature. [R.T.U. 2013]

OR

Define fermi function and fermi energy. Also, explain with graph the variation of fermi function with temperature. [R.T.U. 2015]

Sol.(a) Fermi Energy (E_F)

The fermi energy is a concept of quantum mechanics usually referring to the energy difference between the highest and lowest occupied single particle states in a quantum system of non-interacting fermions at absolute zero temperature.

1. In a system of fermions, such as electrons in a metal, the energy separating the highest occupied single-particle state from the lowest unoccupied one. This definition is not sufficiently precise, however, in many contexts, such as semiconductors, and the term is used (often unwittingly) as a substitute for the chemical potential.
2. The highest filled energy level at absolute zero for a fermion is called the Fermi level. All energy levels below this value are occupied, and all above this value are empty.

Fermi Function

The distribution of electrons (position) in beta decay is calculated from the weak interaction transition matrix using plane wave functions for the electrons. However, electron experience the Coulomb field of the nucleus, so the correct distribution must be modified by the Fermi function, which accounts for this effect.

Fermi Function is given as

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Where E_F is the fermi energy, k is Boltzmann's constant and T is the temperature.

Sol.(b) At $T = 0K$,

for $E < E_F$, $F(E) = 1$ and

for $E > E_F$, $F(E) = 0$

This means that all the energy levels below the Fermi energy, E_F , are completely filled with electrons, whereas all the levels above E_F are vacant.

At $T > 0K$,

for $E = E_F$, $F(E) = 1/2 = 0.5$

At higher temperature, the probability of electrons occupying Fermi energy state is 50%. From this, Fermi Energy can also be defined as the energy at which the probability of occupation of states is 0.5 or 50%. As the temperature is raised, the electrons below E_F get thermally excited and move above E_F to slightly higher energy states in metals.

The thermal energy at a temperature T is kT , where k is Boltzmann's constant.

When $E = E_F - kT$,

$$F(E) = \frac{1}{e^{(-1)} + 1} = 0.73 \rightarrow 73\%$$

of energy states are occupied by electrons. The remaining 27% of energy states below E_F are vacant because the electrons moved to states above E_F . This is verified below.

When $E = E_F + kT$,

$$F(E) = \frac{1}{e^{(1)} + 1} = 0.27 \rightarrow 27\%$$

of energy states are occupied by electrons above E_F . Thus, in an energy span of $2kT$, the probability of occupancy of electrons falls from 73% to 27% around E_F . Fermi energy for metals are about few electron volts. The thermal energy at room temperature 300 K is about 0.025 eV which is much smaller than the Fermi energy for most of the metals.

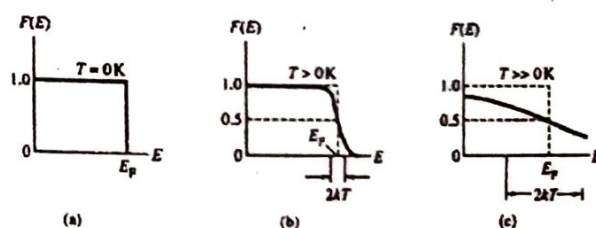


Fig. : Fermi distribution function (a) at absolute zero $T = 0 K$, all the energy states upto E_F are fully occupied (b) at a higher temperature $T > 0 K$ (c) at very high temperature $T \gg 0 K$.

Prob.12 Obtain an expression for conductivity of Intrinsic type of semiconductor. [R.T.U. 2014]

Sol. Intrinsic or Pure Semiconductors

Semiconductors in pure form are called intrinsic semiconductor. For example, a crystal formed by silicon atoms alone. The silicon atoms arrange themselves by sharing an electron between the neighboring atoms using covalent bond.

MODEL QUESTIONS

I. SHORT ANSWER TYPE QUESTIONS

Prob.1 *What is divergence?*

Sol. Divergence : The divergence of a vector field A is its integral over the closed (bound) surface by an infinitesimal volume divided by that volume as the volume shrinks to zero *i.e.*

$$\text{div} A = \lim_{\Delta V \rightarrow 0} \frac{\oint A \cdot ds}{\Delta V}$$

ΔV – bounded volume of the closed surface.

In other words the divergence of A at a given point L is the outgoing flux per unit volume as the volume decreases about L .

In mathematical representation,

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

where A_x , A_y and A_z are the components of vector $\vec{A}$ in x , y and z direction respectively.

Prob.2 What is curl?

Sol. Curl : Let us consider a small contour S centred around the point L of interest such that S encloses a small area element $dS = \hat{n} ds$. The unit vector $\hat{n}$ is in accordance with the sense of traversing the contour S as shown. The curl of $\vec{A}$ at point L is a vector along unit normal and is defined by the relation

$$\vec{\nabla} \times \vec{A} = \text{Curl } \vec{A} = \lim_{dS \rightarrow 0} \frac{\oint \vec{A} \cdot d\vec{l}}{dS} \cdot \hat{n}$$

Prob.3 Find $\nabla \times \vec{r}$.

Sol.

$$\begin{aligned} \nabla \times \vec{r} &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} \\ &= \hat{x} \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z} \right) + \hat{y} \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x} \right) + \hat{z} \left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y} \right) \\ &= 0 \end{aligned}$$

The position vector $\vec{r}$ has no rotational component.

Prob.4 A circular coil is of 10 turns and radius 1m. If a current of 5A flows through it, calculate the field in the coil from a distance of 2m.

Sol. Given : No. of turns $n = 10$, Current $I = 5A$, length $l = 2m$, radius $r = 1m$

The Biot-Savart law formula is given by,

$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi n I}{r}$$

Therefore,

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \times \frac{2 \times \pi \times 10 \times 5}{1} \\ B &= 314.16 \times 10^{-7} T \end{aligned}$$

Prob.5 Write the properties of divergence?

Sol. Properties of Divergence

- (1) It produces a scalar field.
- (2) The divergence of a scalar V , $\text{div } V$ makes no sense.

$$(3) \vec{\nabla} \cdot (\vec{A} + \vec{B}) = \vec{\nabla} \cdot \vec{A} + \vec{\nabla} \cdot \vec{B}$$

- (4) The vector field whose divergence is zero is called solenoid field.

Prob.6 With the field given by $(z - x^2)\hat{i} - y\hat{j} + \sin(x)\hat{k}$, where are there sources and where sinks?

Sol. Since the divergence is $1 - 2x$, it is positive (points are sources of arrows) when $x < 1/2$ and negative (all points are sinks) when $x > 1/2$.

II. LONG ANSWER TYPE QUESTIONS**Prob.1 Derive the Poisson's and Laplace's equation for electrostatic potential?**

Sol. Poisson's and Laplace's Equation for Electrostatic Potential : The relation $\vec{E} = -\vec{\nabla}V$ can also be used when the potential is known throughout the region. Unfortunately, quite often neither the charge distribution nor the potential distribution is known. There is, however, a way out. Given the electrostatic conditions for charge and potential of some boundaries one can solve Laplace's differential equation to obtain $\vec{E}$ and V throughout the region. These are the so-called Boundary-value problems. Laplace's equation appears in such diverse fields of physics as gravitation, magnetism, theory of heat and as many other topics. Our job now is to obtain a suitable equation like Poisson's or Laplace's which satisfy certain boundary conditions.

From the point of view of Gauss's law we have

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

$$\text{with } \vec{D} = \epsilon \vec{E}$$

$$\text{and } \vec{E} = -\vec{\nabla}V$$

$$\therefore \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot (\epsilon \vec{E}) = \vec{\nabla} \cdot (\epsilon \vec{\nabla}V) = \rho_v$$

$$\text{or } \vec{\nabla} \cdot \vec{\nabla}V = \frac{\rho_v}{\epsilon}$$

For a homogeneous region in which ϵ is constant.

For a homogeneous medium we get

$$\nabla^2 V = \frac{\rho_v}{\epsilon}$$

This is the Poisson equation.

That is, a curl-free vector is irrotational. From Stoke's theorem

$$\oint_C (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l} = 0$$

Thus, in an irrotational field $\mathbf{A}$, the circulation of $\mathbf{A}$ around a closed path is identically zero. This implies that the line integral of $\mathbf{A}$ is independent of the chosen path. Therefore, an irrotational field is also known as a *conservative field*. Examples of irrotational fields include the electrostatic field and the gravitational field. In general, the field of gradient ∇ (for any scalar V) is purely irrotational since

$$\nabla \times (\nabla V) = 0$$

Thus, an irrotational field $\mathbf{A}$ can always be expressed in terms of a scalar field V ; that is

$$\text{if } \nabla \times \mathbf{A} = 0$$

$$\text{then } \oint_C \mathbf{A} \cdot d\mathbf{l} = 0 \text{ and } \mathbf{A} = -\nabla V$$

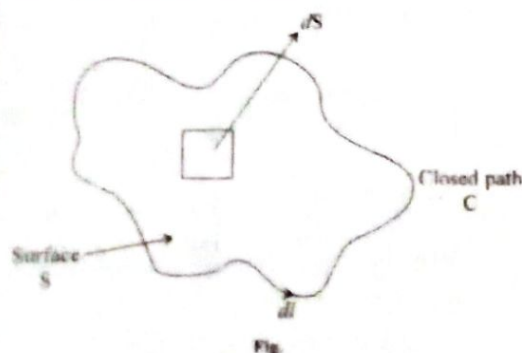
For this reason, $\mathbf{A}$ may be called a potential field and V the scalar potential of $\mathbf{A}$.

Properties of Curl :

- (1) The curl of a vector field is another vector field.
- (2) The curl of a scalar field V , $\nabla \times V$ makes no sense.
- (3) The divergence of the curl of a vector field vanishes, that is $\nabla \cdot (\nabla \times \mathbf{A}) = 0$
- (4) The curl of the gradient of a scalar field vanishes, that is $\nabla \times \nabla V = 0$.

Prob. 8 Explain the Stoke's theorem?

Sol. Stoke's Theorem : Stoke's theorem enables us to transform the surface integral of the curl of the vector field $\mathbf{A}$ into the line integral of that vector field $\mathbf{A}$ over the boundary C of that surface and vice-versa



Statement :

The flux of the curl of a vector function $\mathbf{A}$ over any surface S of any shape is equal to the line integral of the vector field $\mathbf{A}$ over boundary C of that surface i.e.

$$\oint_S \text{Curl } \mathbf{A} \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l} \quad \text{or} \quad \oint_S \nabla \times \mathbf{A} \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l}$$

Proof :

Consider the arbitrary surface as shown in fig. let us divide the surface S bounded by closed curve C into differential elements one of the element is dS . For each such area we form $\oint_C \mathbf{A} \cdot d\mathbf{l}$ taking the contour direction to correspond with the positive sense of dS . If the i^{th} cell has surface area as ΔS_i and is bounded by the path C_i then

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \sum_i \oint_{C_i} \mathbf{A} \cdot d\mathbf{l} = \sum_i \frac{\oint_{C_i} \mathbf{A} \cdot d\mathbf{l}}{\Delta S_i} \Delta S_i$$

there is cancellation of the interior path, so the sum of the line integrals around C_i is the same as the integral around the bounding curve C . Therefore taking $\Delta S_i \rightarrow 0$ and incorporate the definition of curl of a vector, we get

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

which is Stoke's theorem.

Prob. 9 Show that Faraday's law of electromagnetic

induction expressed as $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Sol. Faraday's Law of Electromagnetic Induction :

We know that a current carrying conductor produces a magnetic field. The reverse is also true, i.e., a magnetic field also produces a current, is a very important question. If we have a steady magnetic field near a closed circuit (coil) no current is produced. But if we have a varying magnetic field, this produces a current in the neighboring coil. This may be obtained in several ways :

- (i) Time-varying flux linking a stationary closed circuit / loop.
- (ii) A time-varying loop area in a static magnetic field.
- (iii) A time-varying loop area in a time-varying magnetic field.

The main point is that for a magnetic field to produce a current in a loop, the corresponding magnetic flux must change in some manner.

So, we can write down Faraday's law of electromagnetic induction in the following way.

"An electromotive force (emf) is set up in a circuit when the magnetic flux linking the circuit is changed in some manner; the magnitude of this emf is proportional to the time rate of change of flux linkages with the circuit and the direction of the induced emf is such as tends to oppose the cause".

This means that the direction of the induced emf is such that any current which it produces tends to oppose the change of flux that produces it (Lenz's law).

Faraday's law can now be written as

$$V_{\text{emf}} = -N \frac{d\psi}{dt} \quad \text{---(1)}$$

Where N is the number of turns in the circuit and ψ is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it (Lenz's law).

We consider the cases (i), (ii) and (iii) separately.

(i) **Time-varying flux linking a stationary closed circuit / loop (transformer emf) :** For a circuit with a single turn ($N = 1$) eq. (1) becomes

$$V_{\text{emf}} = -\frac{d\psi}{dt} \quad \text{---(2)}$$

$$\text{and } V_{\text{emf}} = \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\psi}{dt} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S} \quad \text{---(3)}$$

Here S is the surface area of the circuit bounded by the closed path L , and flux per unit turn is given by

$$\psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

One notices from eq. (3) that in a time-varying situation electric and magnetic fields are both present and related to each other. This emf [eq. (3)] is also referred to as transformer emf in power analysis.

So, we get

$$V_{\text{emf}} = \oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \quad \text{---(4)}$$

Using Stoke's theorem we get

$$\oint_C (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \quad \text{---(5)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \dots(6)$$

This is one of Maxwell's equations for time-varying fields.

Since $\nabla \cdot \vec{E} \neq 0$, $\vec{E}$ is not conservative.

Probably Derive the Maxwell's equations in integral and differential form?

Sol. Maxwell's Equations : Maxwell's equations are a set of four partial differential equations that relate the electric and magnetic fields to their sources, charge density and current density. These equations can be combined to show that light is an electromagnetic wave. Individually, these equations are known as Gauss's law for electrostatics, Gauss's law for magnetism, Faraday's law of induction, and Ampere's law with Maxwell's correction. The set of equations is named after James Clerk Maxwell.

These four equations, together with the Lorentz's force law are the complete set of laws of classical electromagnetism.

These equations describe the electric and magnetic fields arising from varying distributions of electric charges and currents, and how those fields change in time. Maxwell's own contribution is just the last term of the last equation, but realizing the necessity of that term had remarkable consequences. It made evident for the first time that varying electric and magnetic fields could feed off each other. These fields could propagate indefinitely through space, far from the varying charges and currents where they originated.

The Maxwell's equations are :

1. Gauss's Law for Electric Fields
2. Gauss's Law for Magnetic Fields
3. Faraday's Law of Magnetic Induction
4. Ampere's Law plus Maxwell's Displacement Current

1. Maxwell's First Equation

Maxwell's first equation is based on Gauss's law for electric field. According to this law, total electric flux passing through a closed surface S in vacuum is equal to the product of total charge (Q) contained inside and $1/\epsilon_0$.

$$\text{i.e. } \oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

for medium

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon} \quad \dots(1)$$

If charge inside the closed surface is continuously distributed and ρ is charge volume density, then we can write

$$Q = \int_V \rho dV \quad \dots(2)$$

From equation (1) and (2)

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon} \int_V \rho dV \quad \dots(3)$$

This is integral form of the Maxwell's I equation.

Using Gauss's divergence theorem

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{E}) dV$$

So, equation (3) can be written as

$$\int_V (\nabla \cdot \vec{E}) dV = \frac{1}{\epsilon} \int_V \rho dV$$

$$\Rightarrow \boxed{\nabla \cdot \vec{E} = \rho / \epsilon} \text{ or } \boxed{\nabla \cdot \vec{E} = \rho / \epsilon} \quad \dots(4)$$

This is differential form of Maxwell's I equation.

If medium is homogeneous and isotropic, then

$$\nabla \cdot (\epsilon \vec{E}) = \rho$$

$$\Rightarrow \boxed{\nabla \cdot \vec{D} = \rho} \quad \dots(5)$$

where $\vec{D} = \epsilon \vec{E}$, is called displacement vector. $\dots(6)$

For free space $\rho = j = 0$

then Maxwell's I equation becomes

$$\oint_S \vec{E} \cdot d\vec{a} = 0 \quad \dots(7)$$

$$\Rightarrow \nabla \cdot \vec{E} = 0 \quad \dots(8)$$

$$\text{or } \nabla \cdot \vec{E} = 0 \quad \dots(9)$$

2. Maxwell's Second Equation

Maxwell's second equation is based on Gauss's law for magnetic field. According to the law, net outgoing magnetic flux from any surface is always zero. This is due to fact that magnetic field lines always from closed loop.

i.e. $\Phi_m = 0$

$$\Rightarrow \oint_S \vec{B} \cdot d\vec{a} = 0 \quad \dots(10)$$

This is integral form of Maxwell's II equation.

Using Gauss's divergence theorem

$$\oint_S \vec{B} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{B}) dV$$

$$\Rightarrow \boxed{\nabla \cdot \vec{B} = 0} \text{ or } \boxed{\nabla \cdot \vec{B} = 0} \quad \dots(12)$$

This is differential form of Maxwell's II equation.

This equation prove theoretically that magnetic monopole does not exist in nature.

3. Maxwell's Third Equation

Maxwell's third equation is based on Faraday's law for electromagnetic induction. According to the law, whenever magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit and this induced e.m.f. is directly proportional to the rate of change of magnetic flux linked with the circuit.

According to Lenz's law, it's direction is always opposite to the change responsible for it's production.

$$\text{i.e. } \vec{e} = -\frac{\partial \Phi_m}{\partial t} \quad \dots(13)$$

Since, line integral of electric field is also equal to the e.m.f.

$$\text{i.e. } \vec{e} = \oint_C \vec{E} \cdot d\vec{r} \quad \dots(14)$$

and we know that magnetic flux

$$\Phi_m = \oint_S \vec{B} \cdot d\vec{a} \quad \dots(15)$$

$$\text{So, } \oint_C \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{a} \quad \dots(16)$$

$$\text{or } \oint_C \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{a} \quad \dots(17)$$

This is integral form of Maxwell's III equation.

Using Stoke's curl theorem

$$\oint_C \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \oint_S (\nabla \times \vec{B}) \cdot d\vec{a} \quad \dots(18)$$

$$\text{So, } \oint_C (\nabla \times \vec{B}) \cdot d\vec{a} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{a} \quad \dots(19)$$

$$\Rightarrow \nabla \times \vec{B} = \frac{\partial \vec{B}}{\partial t} \text{ or } \text{curl } \vec{B} = \frac{\partial \vec{B}}{\partial t} \quad \dots(20)$$

This is differential form of Maxwell's III equation.

For static field (w.r.t. time) i.e. for

$$\frac{\partial \vec{B}}{\partial t} = 0 = \frac{\partial \vec{E}}{\partial t} \quad \dots(21)$$

$$\Rightarrow \nabla \times \vec{E} = 0 \quad \dots(22)$$

$$\text{or } \oint_C \vec{E} \cdot d\vec{r} = 0 \quad \dots(23)$$

4. Maxwell's Fourth Equation

Maxwell's fourth equation is based on Ampere's circuital law. According to the law, line integral of magnetic field over a closed path is equal to n times the net current passes through the closed path.

$$\text{i.e. } \oint_C \vec{B} \cdot d\vec{r} = \mu \Sigma I \quad \dots(24)$$

$$\text{If current density is } \vec{j}, \text{ then } \Sigma I = \oint_S \vec{j} \cdot d\vec{a} \quad \dots(25)$$

$$\text{So, } \oint_C \vec{B} \cdot d\vec{r} = \mu \oint_S \vec{j} \cdot d\vec{a} \quad \dots(26)$$

From stroke's curl theorem

$$\oint_C \vec{B} \cdot d\vec{r} = \mu \oint_S (\nabla \times \vec{B}) \cdot d\vec{a} \quad \dots(27)$$

On comparing equation (26) and (27), we get

$$\nabla \times \vec{B} = \mu \vec{j} \quad \dots(28)$$

But, there is an inconsistency in this law, as taking divergence both side of this equation, we get

$$\nabla \cdot (\nabla \times \vec{B}) = \mu \nabla \cdot \vec{j} \quad \dots(29)$$

But, we know that divergence of curl of a vector is always zero, i.e. $\nabla \cdot (\nabla \times \vec{B}) = 0$

$$\Rightarrow \nabla \cdot \vec{j} = 0 \quad \dots(30)$$

But, from continuity equation

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \dots(31)$$

On comparing equation (30) and (31), we can say that Ampere's law is valid only for steady current, i.e. for which $\rho = 0$ or constant such that

$$\frac{\partial \rho}{\partial t} = 0 \quad \dots(32)$$

We can also say that Ampere's law relates magnetic field to the steady currents. While for the current, produces due to change in electric field w.r.t. time, this law is not valid.

But Ampere's law should be valid for both steady and non-steady currents, otherwise it is contradiction of continuity equation.

Maxwell removed this inconsistency by giving a suggestion that total current density $\vec{J}$ can be considered as made of two type of currents: free current density $\vec{J}_f$, which comes from motion of free charges and displacement current density $\vec{J}_d$, which comes from the change in electric field w.r.t. time.

$$\text{i.e. } \vec{J} = \vec{J}_f + \vec{J}_d \quad \dots(33)$$

equation (7.100) now, becomes

$$\nabla \times \vec{B} = \mu_0 (\vec{J}_f + \vec{J}_d) \quad \dots(34)$$

Again, taking divergence both sides, we get

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 (\nabla \cdot \vec{J}_f + \nabla \cdot \vec{J}_d) \quad \dots(35)$$

$$\Rightarrow \nabla \cdot \vec{J}_f + \nabla \cdot \vec{J}_d = 0$$

$$\nabla \cdot \vec{J}_d = -\nabla \cdot \vec{J}_f \quad \dots(36)$$

But, from continuity equation

$$\nabla \cdot \vec{J}_f + \frac{\partial \rho}{\partial t} = 0 \quad \dots(37)$$

$$\text{or } \nabla \cdot \vec{J}_f = -\frac{\partial \rho}{\partial t} \quad \dots(38)$$

On comparing equation (36) and (38), we have

$$\nabla \cdot \vec{J}_d = \frac{\partial \rho}{\partial t} \quad \dots(39)$$

$$\Rightarrow \nabla \cdot \vec{J}_d = \frac{\partial \rho}{\partial t} (\nabla \cdot \vec{D}) \quad (\text{using Maxwell's 1 equation}) \quad \dots(40)$$

$$\Rightarrow \vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad \dots(41)$$

Thus, after Maxwell's modification, equation (26) becomes

$$\oint_C \vec{B} \cdot d\vec{r} = \mu_0 \oint_S (\vec{J}_f + \vec{J}_d) \cdot d\vec{a} \quad \dots(42)$$

$$\Rightarrow \oint_C \vec{B} \cdot d\vec{r} = \mu_0 \oint_S \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{a} \quad \dots(43)$$

This is integral form of Maxwell's IV equation. using Stoke's curl theorem

$$\oint_C \vec{B} \cdot d\vec{r} = \oint_S (\nabla \times \vec{B}) \cdot d\vec{a} \quad \dots(44)$$

So, equation (40) becomes

$$\oint_S (\nabla \times \vec{B}) \cdot d\vec{a} = \mu_0 \oint_S \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{a}$$

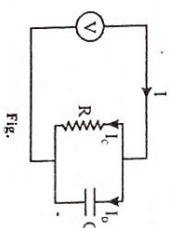
$$\Rightarrow \nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right)$$

$$\text{or } \nabla \times \vec{B} = \mu_0 \vec{J}_f + \mu_0 \frac{\partial \vec{E}}{\partial t} \quad \dots(45)$$

This is differential form of Maxwell's IV equation. It implies that an electric field that changes in time that is current is effective in generating a magnetic field that changes in space.

Prob.11 Explain displacement current in briefly?

Sol. Displacement Current



We consider a parallel combination of a resistance and a capacitor. If we apply voltage V to such a circuit current, a current flowing through resistance is given by

$$I_R = \frac{V}{R}$$

In this case there is actual charge motion and so it is called *conduction current*. Density of this current (conduction current per unit area) is referred to as conduction current density $\vec{J}_f$. The current flowing through the capacitor is given by

$$I_c = \frac{dQ}{dt}$$

$$\text{But } Q = CV$$

$$\therefore I_c = C \frac{dV}{dt}$$

I_c does not imply actual motion of charges. The current flows through the capacitor only when the voltage is changing. The current flowing out from one plate is equal to the current flowing into the other plate. This is displacement current I_d . The displacement current per unit area is called the displacement current density $\vec{J}_d$.

$$\therefore I_d = I_c = C \frac{dV}{dt} \quad \dots(i)$$

Here, C = capacitance of parallel plate capacitor

$$C = \frac{\epsilon A}{d}$$

where A = area of plates

ϵ = permittivity of medium

d = spacing between the plates or thickness of dielectric

E = electric field intensity across the capacitor,

$$\vec{E} = \frac{V}{d}$$

$\therefore$ Eq. (i) gives us

$$I_d = C \frac{dV}{dt} = \frac{\epsilon A}{d} \frac{dV}{dt} = \frac{\epsilon A}{d} \left(d \frac{dE}{dt} \right) = \epsilon A \frac{dE}{dt}$$

But $\vec{D} = \epsilon \vec{E}$

$$\therefore I_d = \frac{d\vec{D}}{dt}$$

$$\text{or } \vec{J}_d = \frac{d\vec{D}}{dt} \quad (\text{displacement current density})$$

The total current density in vector form is given by

$$\vec{J} = \vec{J}_f + \vec{J}_d$$

Prob.12 Write short note on Biot-Savart Law.

Sol. Biot-Savart Law : As we know that magnetic field is produced by the motion of electric charges or electric current. Biot-Savart law gives this relation between current and magnetic field. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current.

Let a certain conductor be carrying a current ' I ' in a direction on the figure. Let ' P ' be the point where the magnetic field due to the wire is to be studied. Let a small portion be considered which is of length ' dl '. Let the line joining ' dl ' and point ' P ' from an angle θ with the tangent to ' dl '.

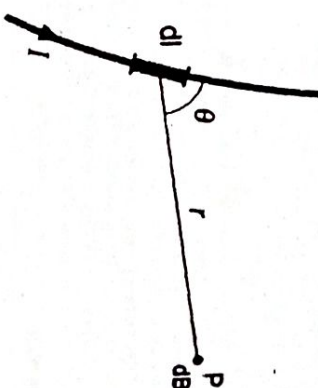


Fig.

Since the portion considered is very small, the magnetic field given by it at point P will also be small. By experimental observations and empirically also, dB is found to depend on several factors.

(i) Here dB is the measurement of magnetic energy which arises from the electrical energy represented by I which act respectively as output and input. Therefore they should have direct dependency i.e.

$$dB \propto I$$

(ii) In a certain length of a conductor, certain amount of charge is present at a moment and the magnetic effect it can produce depend on the total number of charges, which in turn depend on the length consider, i.e.

$$dB \propto dl$$