UNIT – III

PULSE ANALOG MODULATION

SAMPLING THEOREM (About)

Sampling of the signal is the fundamental operation in signal-processing. A continuous time signal is first converted to discrete-time-signal by sampling process

The sufficient number of samples of the signal must be taken so that the original signal is represented in its samples completely.

Also it should be possible to recover or reconstruct the original signal completely from its samples.

The number of samples to be taken depends on max signal freq. present in the signal.

Sampling theorem gives the complete data/Idea About the sampling of signals.

The statement of Sampling Theorem can be given in two parts as:

(i) A band limited signal of finite Energy, which has no freq-component higher than f_m Hz, is completely described by its sample values at uniform intervals less than or equal to $\frac{1}{2f_m}$ second Apart.

(ii) A band-limited signal of finite Energy, which has no frequency components higher than f_m Hz, may be completely recovered from the knowledge of its samples taken at the rate of $2f_m$ samples per second.

The first part represents the representation of the signal in its samples and minimum sampling rate required to represent a continious time signal into its sample.

The second part of the theorem represents construction of the original signal from its samples. It gives sampling rate required for satisfactory reconstruction of signal from its samples.

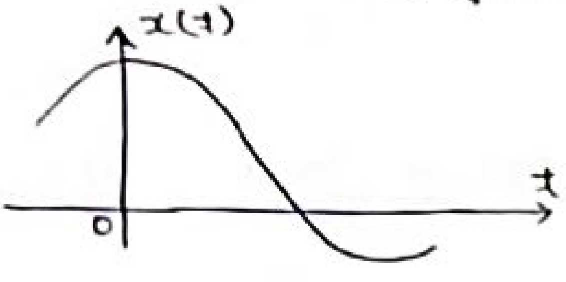
Combining the two Parts :-

"A continious-time signal may be completely represented in its samples and recovered back if the sampling frequency is $\underline{f_s \geq 2 f_m}$."

f_s = Sampling frequency

f_m = Max. frequency in the signal.

Proof of Sampling Theorem



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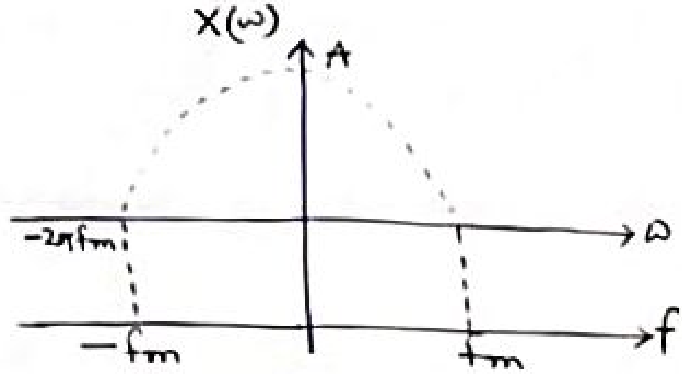

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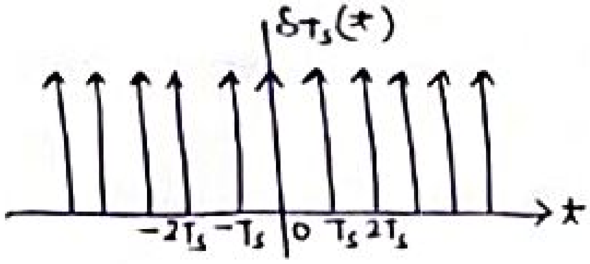

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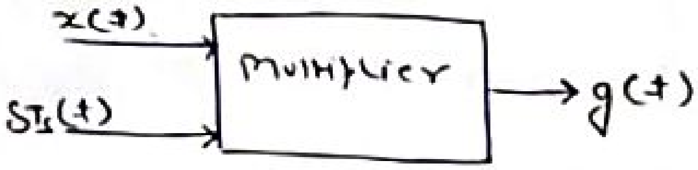


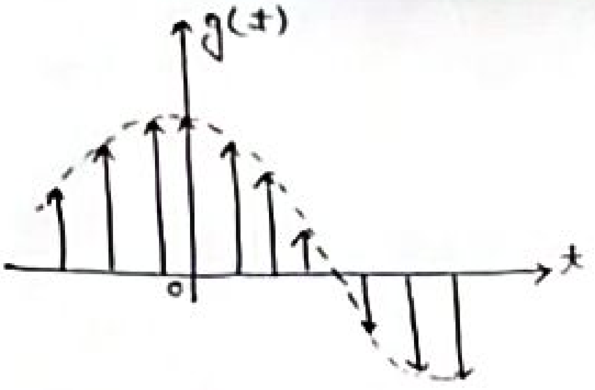
(d)

(a) A continious time signal

(b) Spectrum of contrious time signal

(c) Impule train as sampling function

(d) Multiplier



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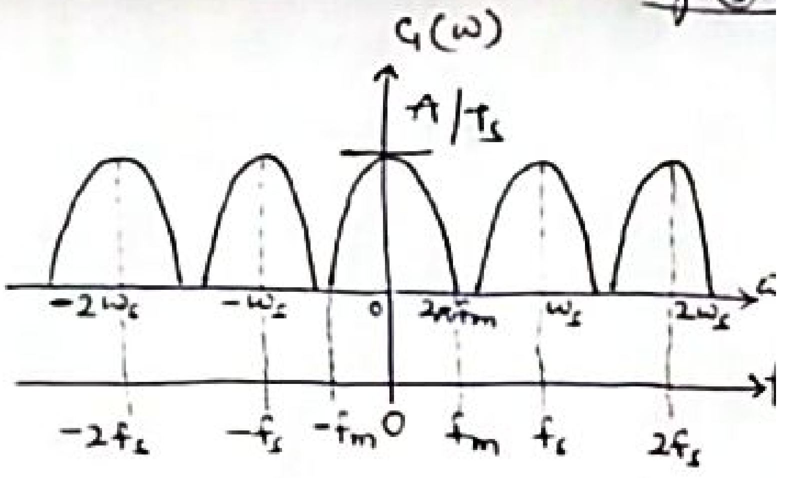

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(e) Sampled signal

(f) Spectrum of sampled signal

Let us consider a continious time signal $x(t)$ whose spectrum is band limited to f_m Hz.

fig. (a) shows thes continious time signal $x(t)$. Let $X(\omega)$ represents its fourier transform of freq. spectrum as fig ⓑ. Sampling of $x(t)$ at a rate of f_s Hz (f_s sample per second) may be achieved by multiplying $x(t)$ by an impulse train $\delta_{T_s}(t)$.

The impulse train $\delta_{T_s}(t)$ consists of unit impulse repeating periodically every T_s second, where $T_s = 1/f_s$. fig ⓒ. The multiplication results in the sampled signal $g(t)$ as shown in fig ⓓ.

This sampled signal consists of impulses spaced every T_s sec. (Sampling Interval). The resulting or sampled signal may be written as,

$$g(t) = x(t)\,\delta_{T_s}(t) \quad \text{-------------- (i)}$$

The impulse train $\delta_{T_s}(t)$ is a periodic signal of period T_s. It may be expressed as a fourier series.

The trignometric fourier series Expansion of Impulse train $\delta_{T_s}(t)$ is Expressed as,

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \ldots\ldots\right] \quad \text{------ (ii)}$$

Here, $\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$

Putting the value of $\delta_{T_s}(t)$ from eq. (ii) in eq. (1), the sampled signal is.

$$g(t) = \frac{1}{T_s}\left[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + 2x(t)\cos 3\omega_s t + \ldots\ldots\right] \quad \text{---- (iii)}$$

Now to obtain $G(\omega)$, the fourier Transformation of $g(t)$ we will have to take the fourier transform of right hand side.

fourier Transform of $x(t)$ is $X(\omega)$.

fourier Transform of $2x(t)\cos\omega_s t$ is $[X(\omega-\omega_s) + X(\omega+\omega_s)]$

fourier Transform of $2x(t)\cos 2\omega_s t$ is $[X(\omega-2\omega_s) + X(\omega+2\omega_s)]$

& so on.

$\therefore$ on taking F.T. the eq. (iii) becomes

$$G(\omega) = \frac{1}{T_s}\left[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + X(\omega-3\omega_s) + X(\omega+3\omega_s) + \ldots\right] \quad \text{---- (iv)}$$

or

$$G(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \quad \text{------------ (v)}$$

from eq. (iv) & (v), it is clear that the spectrum $G(\omega)$ consists of $X(\omega)$ repeating periodically with period $\omega_s = \frac{2\pi}{T_s}$ rad/sec or $f_s = \frac{1}{T_s}$ Hz. (fig f).

Now, if have to reconstruct $x(t)$ from $g(t)$, we must be able to recover $X(\omega)$ from $G(\omega)$ ~~...~~ possible if there is no overlap between successive cycles of $G(\omega)$. This requires

$$f_s > 2f_m \quad \text{------ (vi)}$$

But the sampling interval $T_s = \frac{1}{f_s}$

Hence, $T_s < \frac{1}{2f_m}$ ...... (vii)

∴ as long as the sampling freq. f_s is greater than twice the max. signal frequency f_m, $G(\omega)$ will consist of non-overlapping repetitions of $X(\omega)$.

If this is true, fig (f) shows that $x(t)$ can be recovered from its samples $g(t)$ by passing the sampled signal $g(t)$ through an ideal low-pass filter of bandwidth f_m Hz. This proves the sampling theorem.

Few Points

- It is observed from fig. that for the case $f_s > 2f_m$ the successive cycles of $G(\omega)$ are not overlapping each other. So, there is no problem in recovering the original spectrum $X(\omega)$.

- For the case $f_s = 2f_m$, $G(\omega)$ are not overlapping each other, but they are touching other. In this case also, the original spectrum $X(\omega)$ can be recovered from the sampled spectrum $G(\omega)$ using a LPF with cut-off freq. ω_m

- for the case $f_s < 2f_m$, the successive cycles, of the sampled spectrum will overlap each other & hence in this case, the original spectrum $X(\omega)$ can not be extracted out of the spectrum $G(\omega)$.

NYQUIST RATE & NYQUIST INTERVAL

When the sampling rate becomes exactly to $2f_m$ samples per second, then it is called Nyquist rate. It is also called the minimum sampling rate. It is given by

$$f_s = 2f_m.$$

Similarly, maximum sampling interval is called Nyquist interval. It is given by

$$T_s = \frac{1}{2f_m} \text{ sec.}$$

Numerical

Q.① An analog signal is expressed by the eq. $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$. Calculate the Nyquist rate for this signal.

Q② Find the Nyquist rate & the Nyquist interval for the signal

$$x(t) = \frac{1}{2\pi}\cos(400\pi t)\cos(100\pi t)$$

Q.③ Determine the Nyquist rate for a continious-time signal

$$x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$$

RECONSTRUCTION FILTER (LOW PASS FILTER)

The low pass filter is used to recover original signal from its samples. This is also known as Interpolation filter.

A low pass filter is that type of filter which passes only low frequencies up to a specified cut-off freq. & rejects all other frequencies.

The Interpolation formula

The process of reconstructing a continious-time signal $x(t)$ from its samples is called as Interpolation.

$$\boxed{x(t) = \sum x(kT_s)\,\text{sinc}(2\pi f_m - k\pi)}$$

$$\left\{ \because T_s = \frac{1}{2f_m} \right\}$$

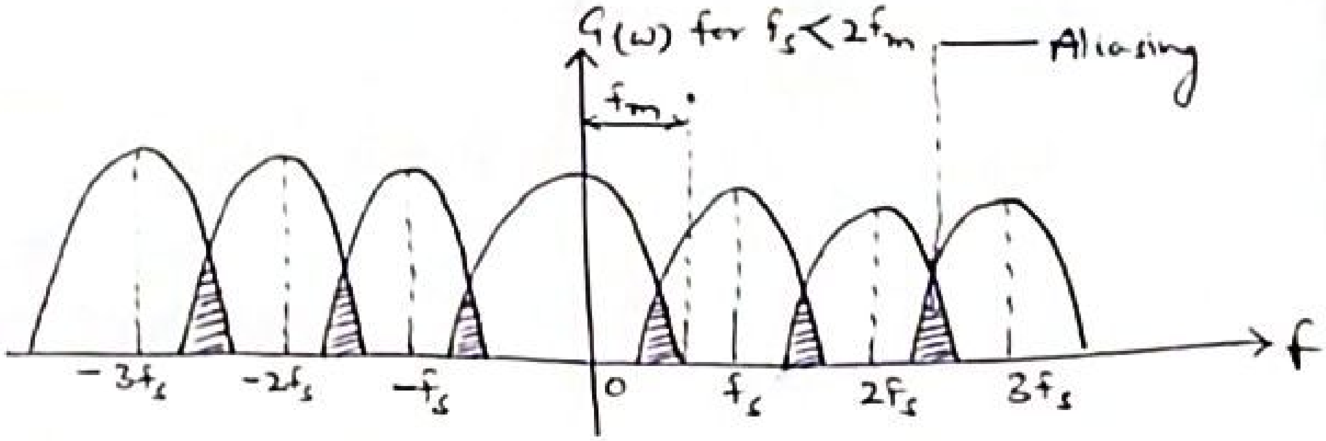
Above equation is known as the interpolation formula, which provides value of $x(t)$ between samples as a weighted sum of all the sample values. In the sampling theorem, it is assummed that the signal $x(t)$ is strictly band limited.

But in general an information signal may contain a wide range of frequencies & can not be strictly band limited.

effect of Under Sampling : Aliasing

when a continious-time-band-limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then the successive cycles of the spectrum $G(\omega)$ of the

Sampled signal g(t) overlap with each other.



Spectrum of the sampled signal for the case $f_s < 2f_m$

Hence, the signal is Undersampled in this case ($f_s < 2f_m$) & some Amount of aliasing is produced in this Under Sampling process.

To Avoid Aliasing

(i) Pre-alias filter (LPF) [Because it is used to prevent aliasing effect] must be used to limit band of frequencies of the signal to f_m Hz.

(ii) Sampling frequency 'f_s' must be selected such that

$$f_s > 2f_m$$

SAMPLING TECHNIQUES :-

Basically there are three types of sampling

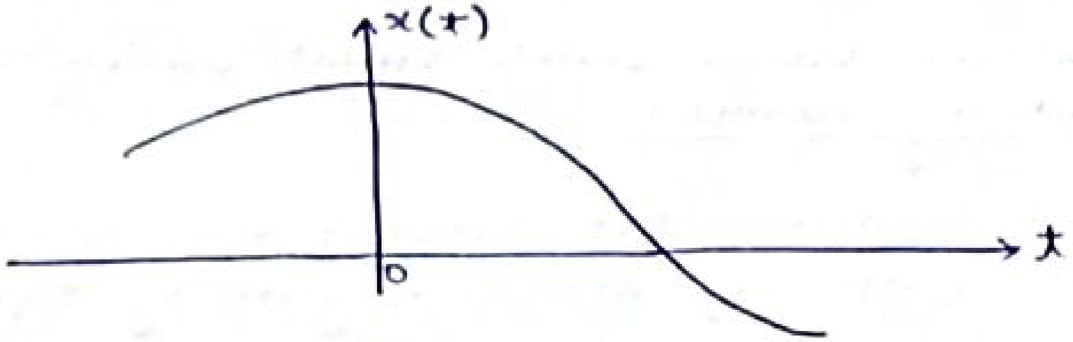
(i) Instentenious sampling

(ii) NATURAL SAMPLING

(iii) Flat top sampling

Out of these three, instantenious sampling is called ideal sampling, whereas natural & flat top sampling are called practical sampling method.

(i) Ideal sampling or Instantenious sampling or Impulse Sampling



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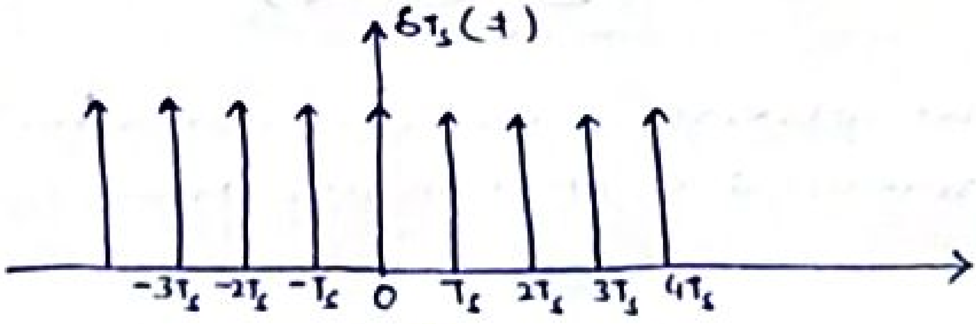

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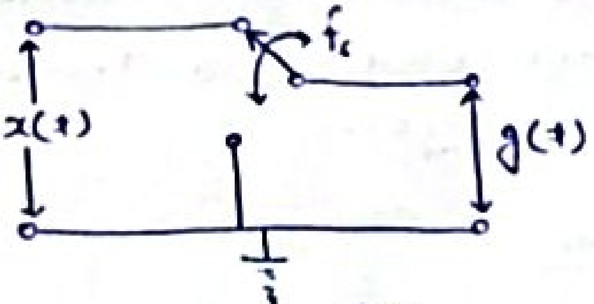

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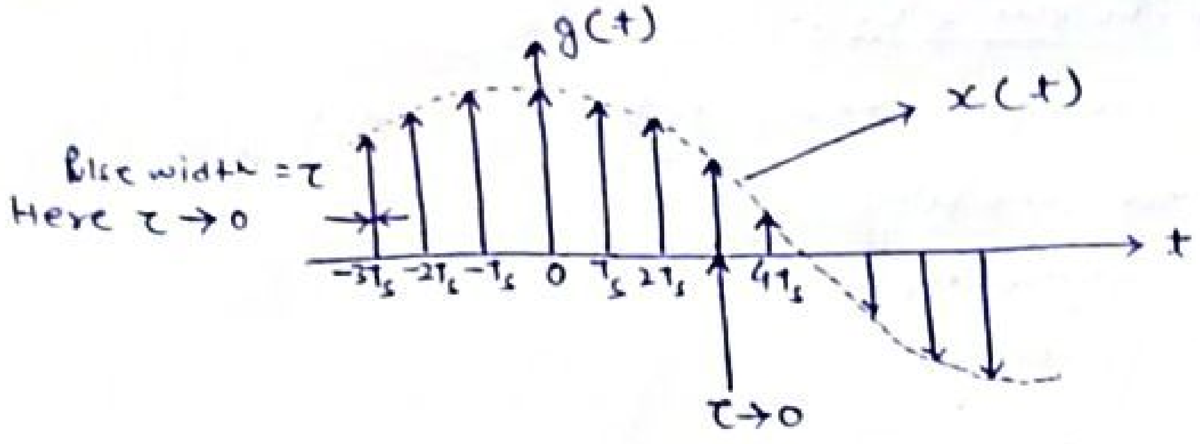


(d)

(a) Base band Signal (b) Impulse Train
(C) functional diagram of a switching sampler
(d) Sampled Signal

This method involves a impulse train to sample analog signal. Here we use a switch circuit which is known as <u>switching sampler.</u>

for ideal condition, the speed of operation is very fast, the output of sample is given by equation.

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Practically we observe some deviation from Ideal sampling. This means the ideal impulse train can not be generated.

There is one more disadvantage of the instantenious sampling that the impulse has width τ Approaching to zero, means not useful for Transmission because its power content is zero.

NATURAL SAMPLING :- https://ersahilkagyan.com

As discussed in Last Article, the instantenious sampling results in the samples whose width τ approached to zero Due to this, the power content in the Instantenious

Thus this method is not suitable for Transmission purpose.

Natural sampling is a practical method. In natural sampling the pulse has a finite width equal to τ.

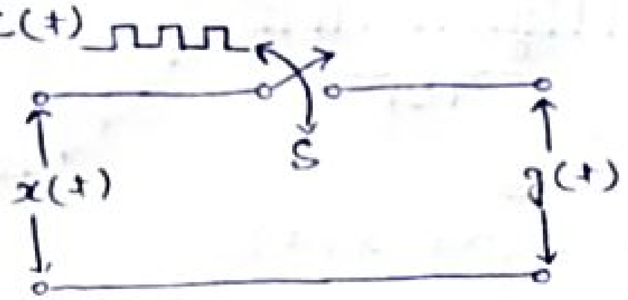
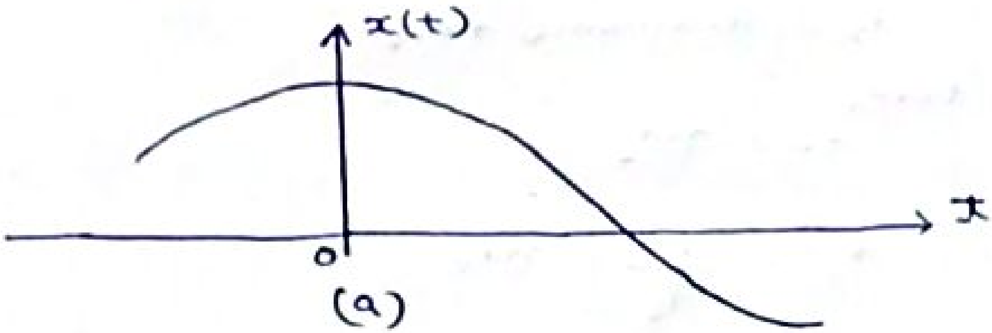
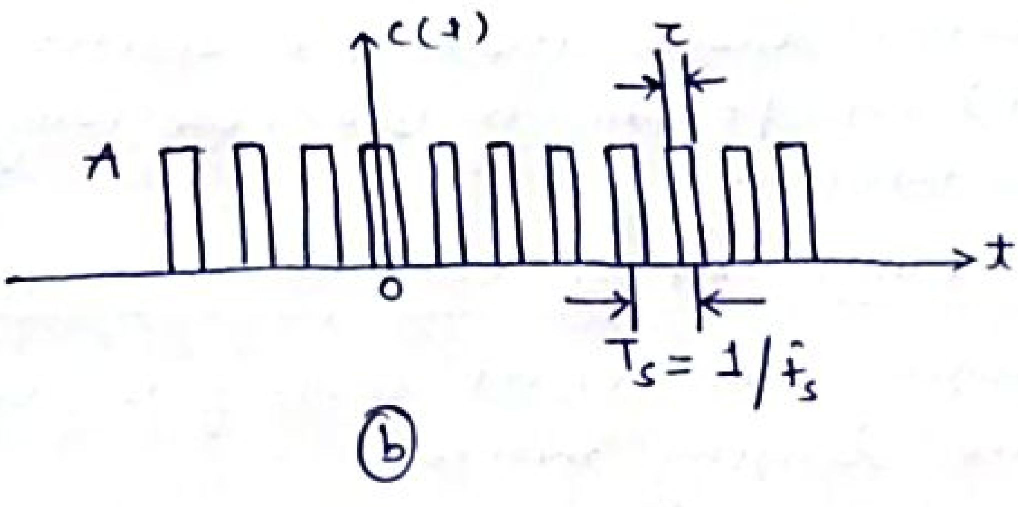
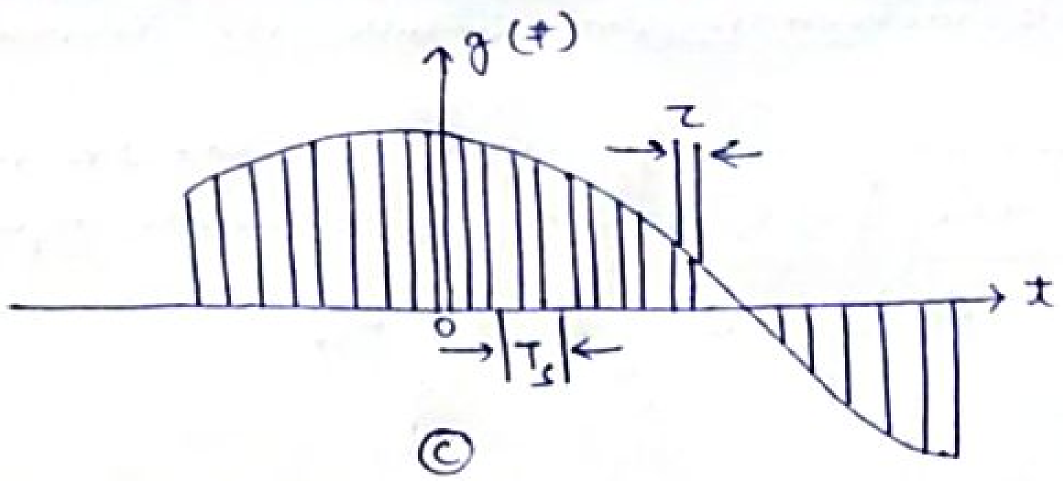


fig. A functional diagram of a Natural sampler.

In this practical method, we consider that the impulse train is of finite duration τ, and a analog signal $x(t)$ is applied to a switching ckt. controlled by sampling signal $c(t)$. which having an infinite succession of pulses of width τ and amplitude A.







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(a) Continious time signal $x(t)$
(b) Sampling function waveform or periodic pulse train.
(c) Naturally sampled signal waveform $g(t)$

Spectrum of Naturally sampled signal

$$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(nf_s \cdot \tau)\, X(f - nf_s)$$

here, T = pulse width = τ

f_n = harmonic freq.

But here, $f_n = nf_s$

or

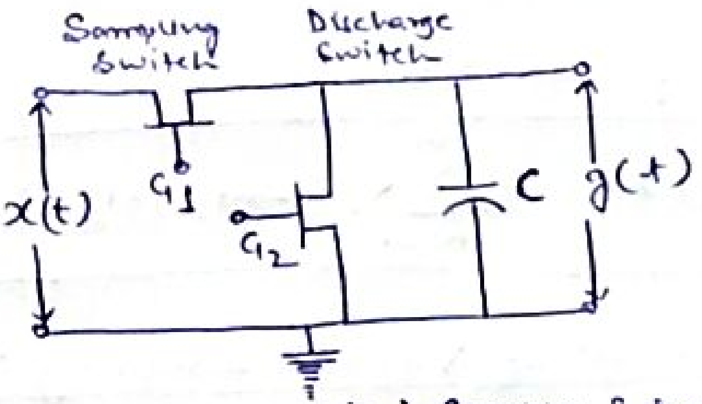
$$f_s = \frac{n}{T_0} = nf_0$$

Above equation shows that the spectra of $x(t)$ i.e.. $X(f)$ are periodic in f_s and are weighed by the sinc function.

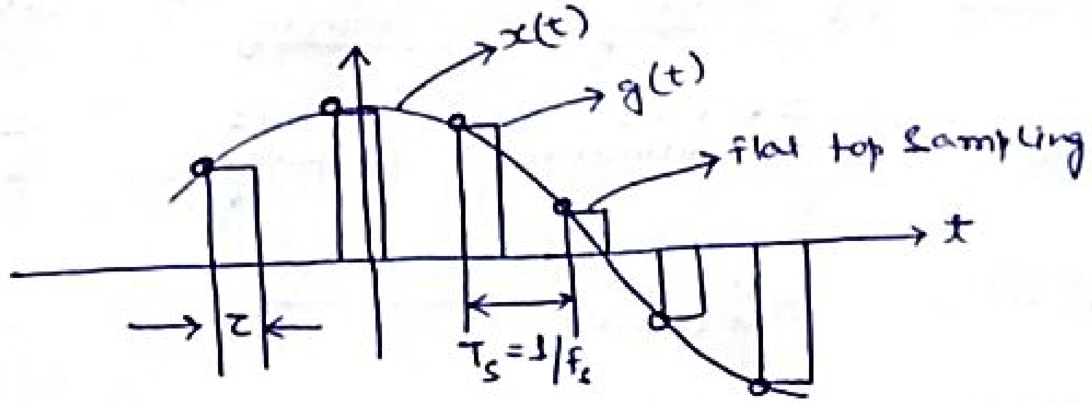
FLAT TOP SAMPLING or RECTENGULAR PULSE SAMPLING

flat top sampling like natural sampling is also a practically possible sampling method.

But natural sampling is little complex. In flat-top sampling or rectengular pulse sampling, the top of the samples remains constant & is equal to the instantenious value of the baseband signal $x(t)$ at the start of the sampling. The duration or width of each sample is τ & sampling rate is equal to $f_s = \frac{1}{T_s}$



(a) Sample & hold Ckt.



(b) A General wave form of flat top sampling.

fig (a) shows the functional diagram of a sample & hold ckt. which is used to generate the flat top samples. from fig (b), it may be noted that only starting edge of the pulse represents instantenious value of the baseband signal $x(t)$.

The sample & hold ckt. consist of two FET switch & a capacitor.
when a pulse train is applied to the gate G_1 of transistor the switch is closed, the capacitor is charged

up to voltage equal to the instantenious sampl value of the coming signal it hold the charg until the discharging pulse is Applied to G_2.

Here the capacitor hold the charge so the capaci tor voltage doesn't follow the shape of Applied signal.

The spectrum of flat top sampled signal

$$\boxed{G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) H(f)}$$

Performance Comparison of Various Sampling techniques.

Parameter	Ideal Sampling	Natural Sampling	flat top Sampling
Sampling Principle	It uses multiplication	It uses Chopping.	It uses Sam & hold ckt.
Generation ckt.	Already discussed	→	→
waveform Involved	"	→	→
feasibility	This is not a practically possible method	Practically used	Practically used.
Noise Interference	Max.	min.	Max.
freq. domain Representation	equation discussed	→	→

Analog Pulse Modulation Methods

In Pulse modulation methods, the carrier is no longer a continious signal but consists of a pulse train.
Some parameter of which is varried according to the instantenious value of the modulating signal.
There are two types of Pulse modulation system as under

(i) Pulse Amplitude Modulation (PAM)
(ii) Pulse Time Modulation (PTM)

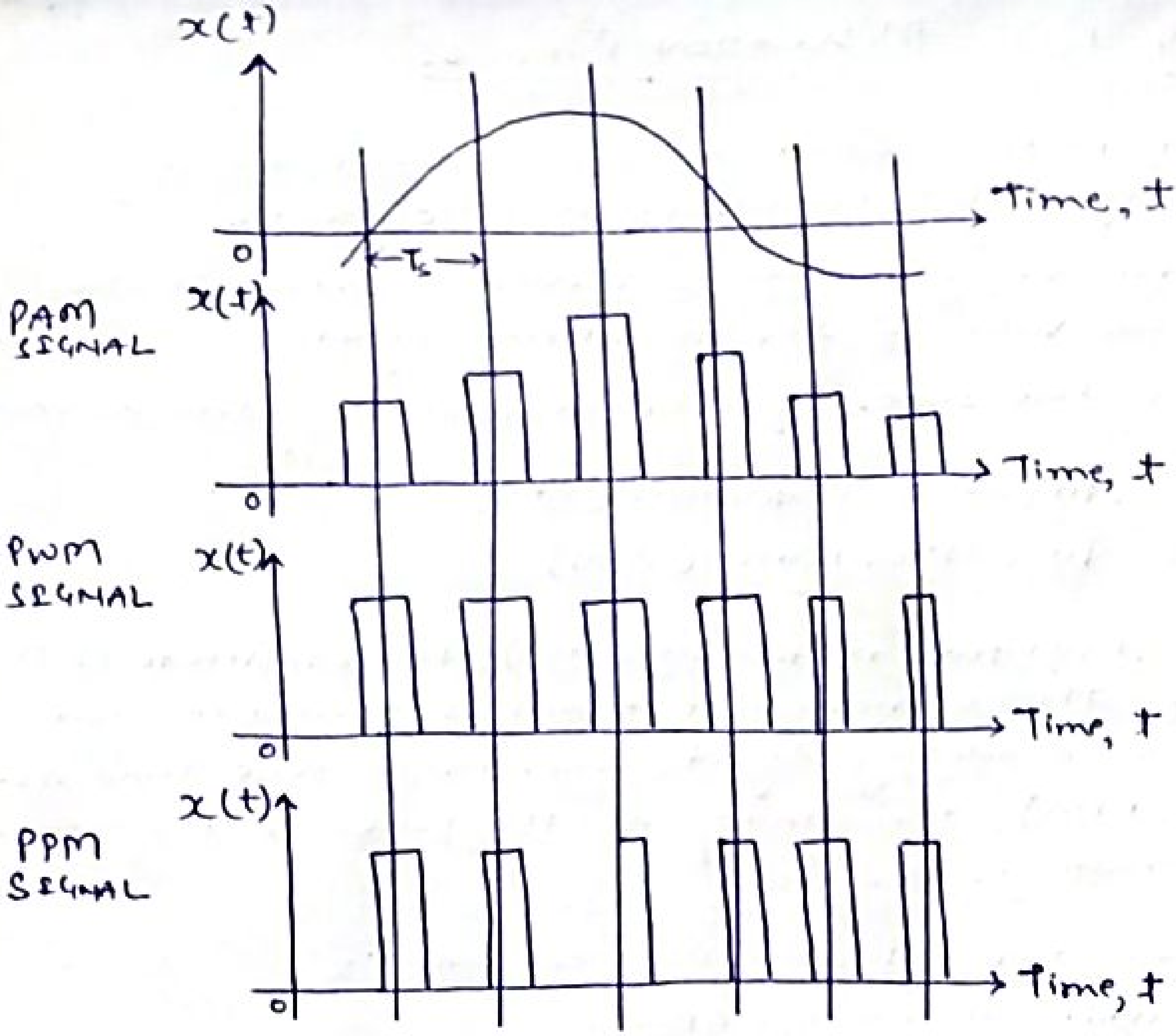
In Pulse Amplitude modulation (PAM), the amplitude of the pulses of the carrier pulse train is varried in accordance with the modulating signal, whereas in pulse Time modulation (PTM), the timing of the pulses of the carrier pulse train is varried.

There are two types of PTM as under:-
(i) Pulse width modulation (PWM)
or
Pulse Duration Modulation (PDM)

(ii) Pulse Position Modulation (PPM)

In Pulse width modulation, the width of the pulse of of the carrier pulse train is varried in accordance with the modulating signal.
In Pulse Position Modulation, the Position of the pulses of the carrier pulse train is varried.

All the Above pulse modulation method (PAM, PWM, PPM) are called analog pulse modulation methods, because the modulating signal is analog in nature.



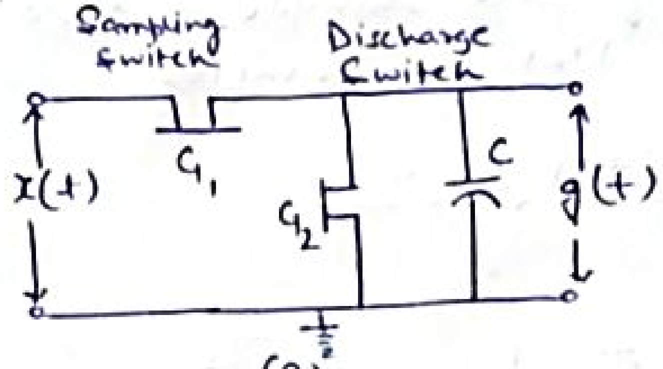
Different types of Pulse analog modulation Methods.

Pulse Amplitude Modulation (PAM) (Generation/Modulation)

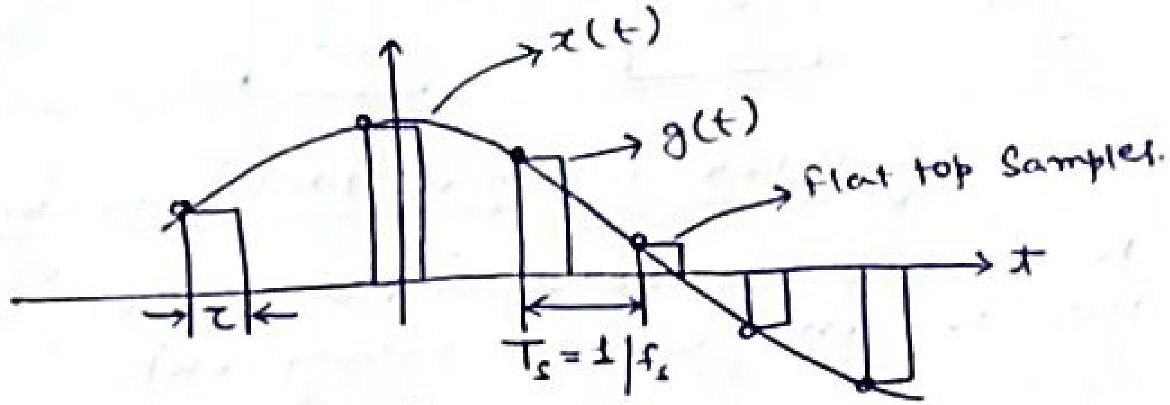
Pulse Amplitude modulation may be defined as that type of modulation in which the amplitudes of regularly spaced rectengular pulses vary according to instanteni-ous value of the modulating or message signal.

The flat top PAM is most popular & is widely used. The reason for using flat top PAM is that during the Tra-nsmission, the noise interferes with the top of the transmitted pulses & this noise can be easily remo-ved if the PAM pulse has flat Top.

The fig. shows the Sample & hold circuit to produce flat top sampled PAM & the waveform for flat top sampled PAM.



(a)



(b)

Working Principle :-

The working principle of this ckt is quite easy. The sample & hold (S/H) ckt consists of two field effect Transistor (FET) switches & a capacitor.

The sampling switch is closed for a short duration by a short pulse applied to the Gate G_1 of the Transistor.

During this period the capacitor C is quickly charged up to a voltage equal to the instantenious sample value of the incoming signal x(t).

Now the sampling switch is opened & the capacitor C holds the charges.

The discharge switch is then closed by the pulse applied to the gate G_2.

Due to this the capacitor 'C' is discharged to zero volts

The discharge switch is then opened & thus capacitor has no voltage.

Hence the o/p of the sample & hold ckt. consists of a sequence of flat top samples.

Demodulation of PAM signals.

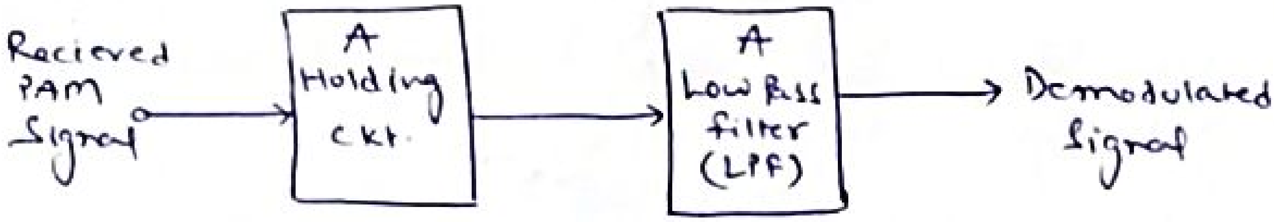
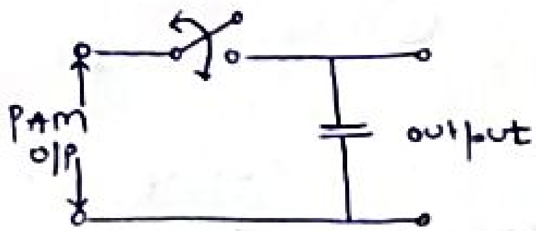
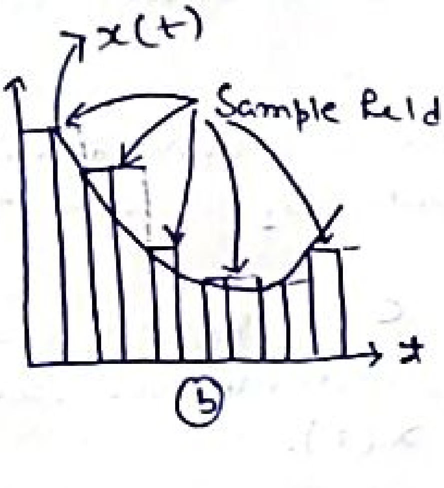


fig. Block diagram of PAM Demodulator.

for Pulse-Amplitude modulated (PAM) signals, the demodulation is done using a holding ckt.



(a)



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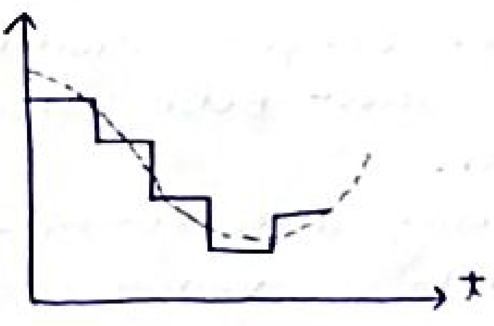


(c)

(a) A zero-order holding ckt.

(b) The output of holding ckt.

(c) The output of a LPF.

In this method, the recieved PAM signal is allowed to pass through a holding ckt. & a low pass filter (LPF).

The ckt. in fig (a) is known as <u>zero-order Holding ckt.</u> This zero-order holding ckt. considers only the previous sample to decide the value between the two points.

<u>TRANSMISSION OF PAM SIGNALS</u>

If the PAM signals are to be transmitted directely over a pair of wires then no further signal processing is necessary.

However if they are to be transmitted through the space using an antenna, they must be first amplitude or frequency or phase modulated by a high freq. carrier & only then they can be transmitted.

Thus the overall system will be then known as PAM-AM or PAM-FM or PAM-PM respectively.

At the recieving end, AM or FM or PM detection is first employed to get the PAM signal & then the message signal is recovered from it.

<u>DRAWBACKS</u> :-

(i) Bandwidth requirements for the PAM Tx is very large.

(ii) Interference of noise is maximum in a PAM signal. This noise cannot be removed easily.

<u>PULSE TIME MODULATION</u> :-

In pulse time modulation, the signal to be transmitted is sampled as in pulse amplitude modulation (PAM).

In pulse time modulation, amplitude of pulse is held constant, wheras position of pulse of width of pulse is made proportional to the amplitude of signal at the sampling instant.

As we know that PTM can be classified in two type.

① PWM ② PPM.

Pulse width modulation (PWM)

This is also known as pulse duration modulation (PDM).

Three variations of pulse width modulation are possible.

In one variation, the lending edge of the pulse is held constant & change in pulse width with signal is measured with respect to the lending edge.

In other variation, the tail edge is held constant & with respect to it, pulse width is measured.

In the third variation, centre of the pulse is held constant & pulse width changes on either side of the centre of the pulse.

MODULATION of PWM signal (Generation)

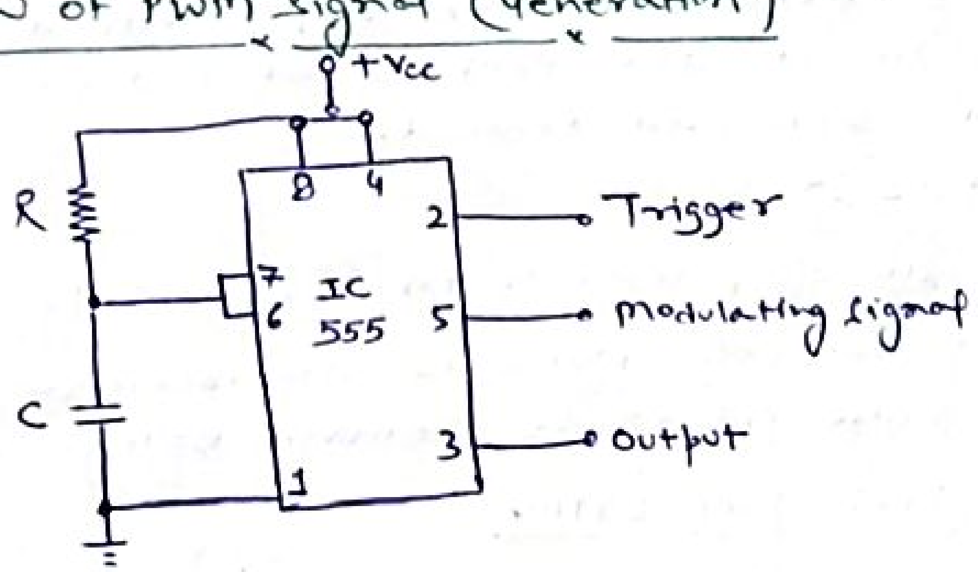


fig. shows pulse width modulator, It is basically a monostable multivibrator with a modulating input signal applied at the control voltage input. Internally, the

Control voltage is adjusted to the 2/3 V_{CC}.
Externally applied modulating signal changes the Control voltage, & hence the threshold voltage level. As a result, the time period required to charge the Capacitor up to threshold voltage level changes, giving pulse modulated signal at the output.

Demodulation of PWM signal

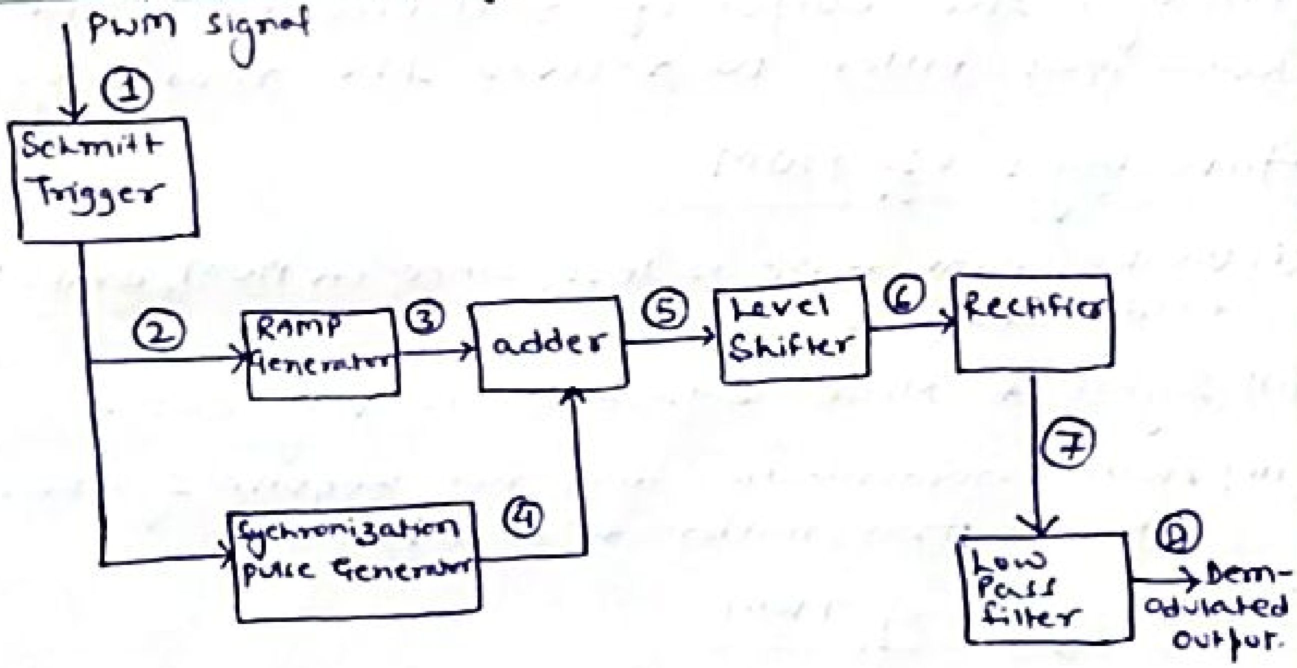


fig. PWM Detector

The recieved PWM signal is applied to the Schmitt trigger ckt.
The Schimitt trigger ckt removes the noise in the PWM waveform.
The regenerated PWM is then applied to the ramp generator & the synchronization pulse detector.
The ramp generator produces ramp for the duration of pulses such that height of ramps are proportional to the widths of PWM pulses.
The max. ramp voltage is retained till the next pulse.

on the other hand, Synchronous pulse detector produces reference pulses with constant Amplitude & pulse width. These pulses are delayed by specific amount of delay. The delayed reference pulses and the output of ramp generator is added with the help of adder.

The o/p of Adder is given to the level shifter. Then the negative part of the waveform is clipped by rectifier.

finally, the output of rectifier is passed through Low-pass filter to recover the modulating signal.

Advantages of PWM

(i) Unlike, PAM, noise is less, since in PWM, amplitude is held constant.

(ii) Signal & Noise Seperation is very easy.

(iii) PWM communication does not require synchronization between transmitter & Reciever.

Disadvantages of PWM

(i) Large Bandwidth is required for the PWM communication as compared to PAM.

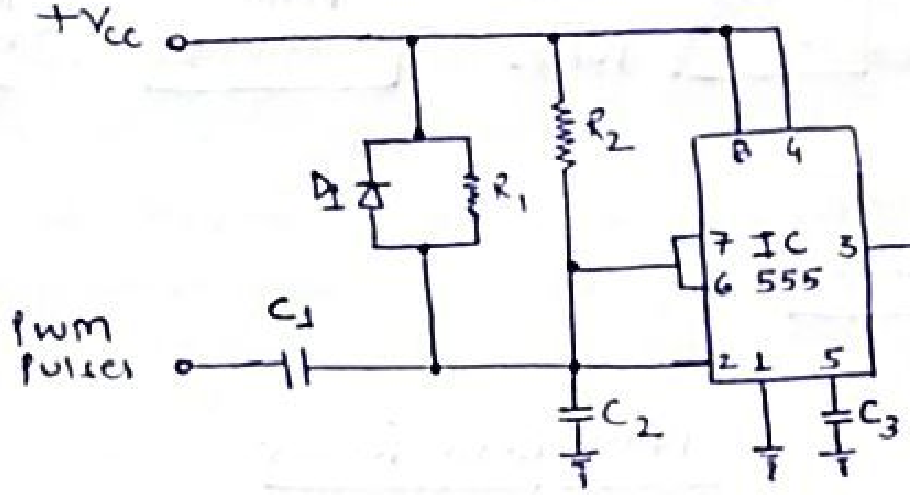
PULSE POSITION MODULATION

In this system, the amplitude & width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse, is changed according to the intentenious sampled value of the modulating signal.

Thus the transmitter has to send synchronizing pul to keep the transmitter & Reciever in synchronism

Pulse Position modulation is obtained from pulse width modulation.

Generation of PPM signal



PPM Generator

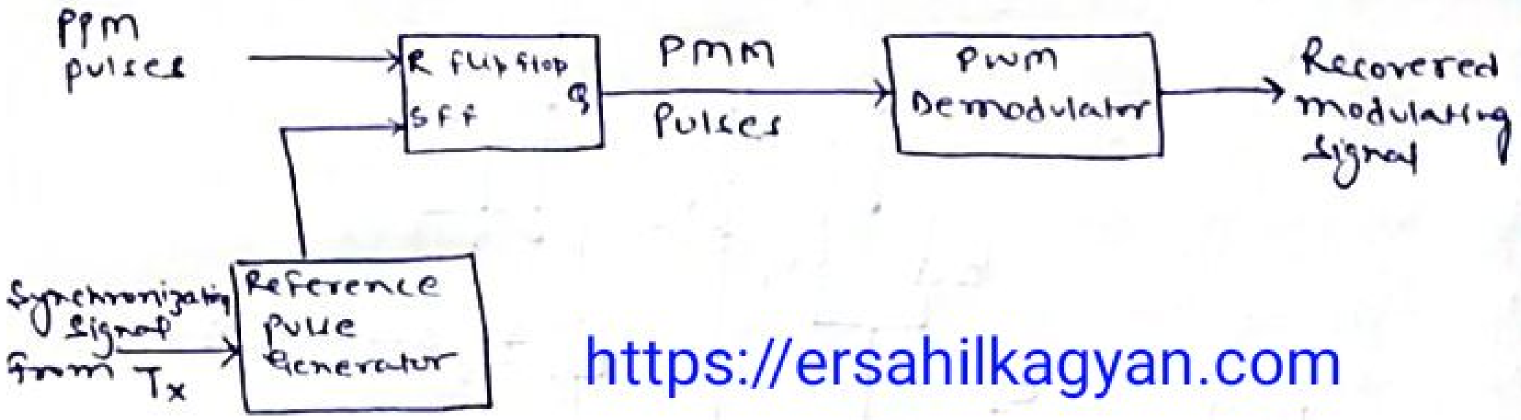
Above fig. shows the PPM generator. It consists of differentiator & and a monostable multivibrator. The input of the differentiator is a PWM waveform. The differentiator generates positive & negative spikes corresponding to leading & trailing edges of the PWM waveforms.

Diode D_1 is used to bypass the positive spikes. The negative spikes are used to trigger monostable multivibrator.

The monostable multivibrator then generate the pulses of same width & amplitude with reference to trigger to give pulses position modulated waveform.

Demodulation of PPM

In case of pulse-position modulation, it is customary to convert the recieved pulses that vary in position to pulses that vary in length.



PPM Demodulator

As shown in fig., flip-flop ckt. is set or turned 'ON' (giving high output), when the reference pulse arrives.

This reference pulse is generated by reference pulse generator of the reciever with the synchronization signal from the transmitter.

The flip-flop circuit is reset or turned 'OFF' (giving low output) at the leading edge of the position modulated ~~[illegible]~~ pulses.

This repeats & we get PWM pulses at the O/P of the flip-flops.

The PWM pulses are then demodulated by PWM demodulator to get original modulating signal.

Advantages of PPM

(i) Less noise interference.
(ii) Signal & Noise seperation is very easy.
(iii) Transmission power for each pulse is same.

Disadvantages of PPM

(i) Synchronization between Tx & Rx is required.
(ii) Large bandwidth is required as compared to PAM.